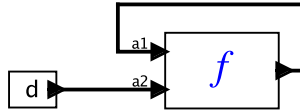


## Exercises

1. Suppose  $A = \{\perp\} \cup \mathbb{N}$  is a flat CPO with  $\perp$  at the bottom and each element of  $\mathbb{N}$  immediately above  $\perp$ . Each element of  $\mathbb{N}$  is incomparable with all other elements of  $\mathbb{N}$ . Consider a function  $f: A^2 \rightarrow A$  given by, for all  $a_1, a_2 \in A$ ,

$$f(a_1, a_2) = \begin{cases} \perp & \text{if } a_1 = a_2 = \perp \\ a_1 & \text{if } a_1 \in \mathbb{N} \\ a_2 + 1 & \text{otherwise} \end{cases}$$

- (a) Is this function strict?  
 (b) Suppose this function is used in the following SR model:



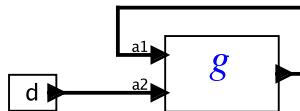
where  $d = 1$  is produced on every tick by the left actor. Find all possible outputs of  $f$ . Is there a least fixed point?

- (c) Show that this function is not monotonic.  
 2. Suppose  $D$  is a set of data values and  $A = \{\perp, \text{absent}\} \cup D$  is a flat CPO with  $\perp$  at the bottom. Consider a function  $g: A^2 \rightarrow A$  given by, for all  $a_1, a_2 \in A$ ,

$$g(a_1, a_2) = \begin{cases} \text{absent} & \text{if } a_1 = a_2 = \text{absent} \\ d & \text{if } a_1 \in D \text{ or } a_2 \in D \\ \perp & \text{otherwise} \end{cases}$$

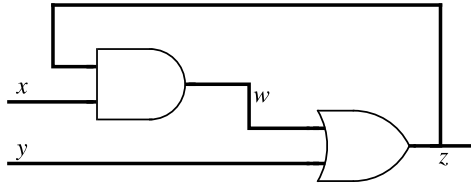
for some  $d \in D$ .

- (a) Is this function strict?  
 (b) Is this monotonic? continuous?  
 (c) Is this function sequential?  
 (d) Suppose this function is used in the following SR model:



where  $d \in D$  is produced on every tick by the left actor. What is the output of  $g$  on each tick? What if  $d$  is replaced with *absent* in the source actor on the left? Assume the constructive semantics.

3. The following questions relate to the following circuit diagram, which should be interpreted under synchronous semantics:



- Is this circuit sequential or combinational? Explain.
- For input  $(x, y) = (0, 1)$ , is the circuit constructive? Explain.
- For input  $(x, y) = (1, 0)$ , is the circuit constructive? Explain.
- Perform symbolic execution, and find the characteristic functions for nodes  $w$  and  $z$  as a function of  $x$  and  $y$ .
- Use your result from part (d) to show whether the circuit is constructive.