Exercises

1. Consider the model shown in figure 6.2.

   (a) Prove that the signal s (into the TimedPlotter) is not discrete.

   (b) Construct a model in the DE domain in Ptolemy II similar to the one in figure 6.2 with the following properties:

      i. it has a feedback loop where no actor is delta causal,

      ii. time diverges (it has events at times greater than any finite time), and

      iii. the system is discrete (there is no Zeno condition).

   This demonstrates that the condition requiring a delta-causal actor in a feedback loop is only sufficient, not necessary, to prevent Zeno conditions.

2. Consider an actor $F\colon (T \rightharpoonup V) \to (T \rightharpoonup V)$ for tag set $T = \mathbb{R}_+$ and value set $V = \{0\}$, a singleton set, where for the n^{th} input event $(t_n, 0)$, there is an output event $(t_n + \tau_n, 0)$, where $0 < \tau_n < 1$. Specifically, let

$$\tau_n = 1/(n+1), \quad n \in \mathbb{N}.$$

   Determine whether each of the following statements is true or false and give a justification.

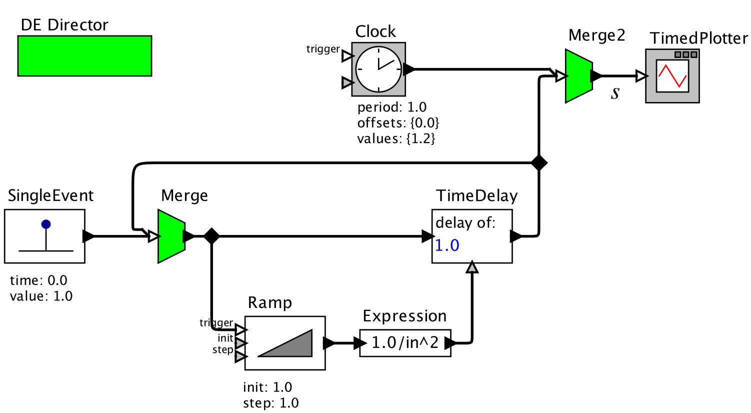

Figure 6.2: A discrete-event model that exhibits Zeno behavior.

(a) F is discrete.

(b) F is monotonic in the prefix order.

(c) F is strictly causal under the Cantor metric.

3. This problem explores the structure of the metric space that is the set of discrete signals $(\mathbb{R}_+ \rightharpoonup \mathbb{R})$ with the Cantor metric d.

   (a) An **open neighborhood** around a signal $s \in (\mathbb{R}_+ \rightharpoonup \mathbb{R})$ is a set of signals given by
   $$N(s,r) = \{s' \in (\mathbb{R}_+ \rightharpoonup \mathbb{R}) \mid d(s,s') < r\},$$
   for some real number r (called the **radius**). For the Cantor metric, describe in words the set $N(s,r)$ for $r = 0.5$ and s given by its graph
   $$\mathcal{G}(s) = \{(1,1)\}.$$

   (b) Show that this metric space is **Hausdorff**,* meaning that given any $s_1, s_2 \in (\mathbb{R}_+ \rightharpoonup \mathbb{R})$ such that $s_1 \neq s_2$, there are open neighborhoods U_1 and U_2 in $(\mathbb{R}_+ \rightharpoonup \mathbb{R})$ such that $s_1 \in U_1$, $s_2 \in U_2$, and $U_1 \cap U_2 = \emptyset$.

*The significance of this property of a metric space can be understood with the mnemonic "housed off." In such a metric space, given two distinct elements, it is always possible to construct non-overlapping open-neighborhoods around each element. That is, each element can be "housed off" from the other.