Lecturer: Prof J. Harrison (Office: 829 Evans Hall)
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Lecture times: MWF 11:00-12:00 740 Evans

Office hours: MWF 1:00-2:00

Assessment: 100% final examination, mostly based on homework

Algebraic topology is a branch of mathematics that uses tools from abstract algebra to study topological spaces. The basic goal is to find algebraic invariants that classify topological spaces up to homeomorphism or homotopy equivalence. We will construct three such invariants: the fundamental group, homology groups, and the cohomology ring. We will use these to prove classical results such as the classification of surfaces, the Brouwer fixed point theorem, the Jordan curve theorem, the Lefschetz fixed point theorem, and more.

An important topic related to algebraic topology is differential topology, i.e. the study of smooth manifolds. We will cover the basics of the topology of manifolds, in order to provide more intuition and applications.

Prerequisites: It helps to have already come across the concepts of "continuity" for maps between topological spaces or metric spaces, and "compactness" for subsets of such spaces. (See Rudin, Principles of Real Analysis.) Familiarity with concepts of elementary algebra, including abelian groups, quotient spaces, direct sums is essential. I will also assume a working background in abstract linear algebra (see my notes, Universal Linear Algebra for Students of Math and Physics.) Our main text follows a functorial approach and it will be helpful to understand the basic definitions of a category, functors, natural transformations, and universal properties.

1. Homotopy

Fundamental groups, covering spaces, homotopy equivalence, functoriality, CW complexes.

The fundamental group and covering spaces.

2. Homology

Homology of CW complexes, singular homology, homological algebra, relative homology. Mayer-Vietoris sequence and computations. Kunneth formula, Homology with coefficients.

3. Cohomology theory Cech cohomology and de Rham cohomology. Equivalence between singular, Cech and de Rham cohomology, Poincare and Alexander duality

4. Products and Lefschetz Fixed point theorems

There will be exercises assigned each week. Hopefully we will have a grader, but the marks will not count towards your overall score.

Texts:

- J J Rotman, An Introduction to Algebraic Topology (Springer GTM 119, 1988) Rotman is tempting as the book of choice for our course. I just learned about it and am making a decision now. I will shortly update the syllabus accordingly (Aug 25, 2015.)
- C R F Maunder, Algebraic Topology (Van Nostrand 1970) You can get a used Dover edition on amazon for $10, including shipping. We will use this for our study of homotopy. Everyone should read his first chapter, a crash course on algebraic and topological preliminaries. It is well written and forms the backbone of the course. By Monday Sept 1, I will assume you have read it and are familiar with its notation and constructions.
- M J Greenberg and J Harper, Algebraic Topology: a First Course (Benjamin/Cummings 1981). Greenberg and Harper is quite concise, a la Rudin, and takes a functorial approach which is ideal for our course. This has an excellent treatment of homology and cohomology and emphasizes the axiomatic point of view with a touch of category theory. It went out of print some time ago, and thus fell into disuse, but there is now a pdf copy available.
- G E Bredon, Topology and Geometry (Springer GTM 139, 1993). This is a good reference for differentiable manifolds. A pdf is available, but I need to see if it is ok for me to give you a copy.
- A Hatcher, Algebraic Topology (Cambridge University Press, paperback, 2002). Hatcher's book provides plenty of additional reading for those who want to follow the subject further. The CUP paperback edition is not expensive, but the book is also available on Hatcher's web-page and copies are permitted for non-commercial use. He motivates the topics very well with nice pictures and descriptive language. His definitions are not always clearly stated, his proofs sometimes lack rigor and are not especially concise. Since he provides a free pdf, we can use it as a reference to help with your intuition and give you clear geometrical insights. I might assign some homework problems from Hatcher.

Other outstanding books you should know about:

- E H Spanier, Algebraic Topology (McGraw Hill 1966) Spanier is the comprehensive text on algebraic topology, but it is very formal in style and difficult to read if one is not already familiar with the basic ideas. If you want to become an algebraic topologist, per se, then this is a book you must read.
- S. Eilenberg & N. Steenrod, Foundations of Algebraic Topology (1954) This is a classic and remarkably beautiful. I really wanted to use this, but it is a bit old fashioned and does not cover CW complexes. This is the book that taught the world about the celebrated Eilenberg Steenrod axioms. It followed on the heals of Eilenberg & Mac Lane's papers on Category Theory (1942-45.) Their work was central to the transition from intuitive geometric homology to axiomatic homology theory. Their initial goal was to understand natural transformations. For this, functors had to be defined, and thus categories.
- R Bott and L W Tu, Differential Forms in Algebraic Topology (Springer GTM 82, 1982). This one is a treasure. It is a blend of differential and algebraic topology and has a wealth of information. Unfortunately, we rarely use this as a text at Berkeley since it blends two courses, 214 and 215A.