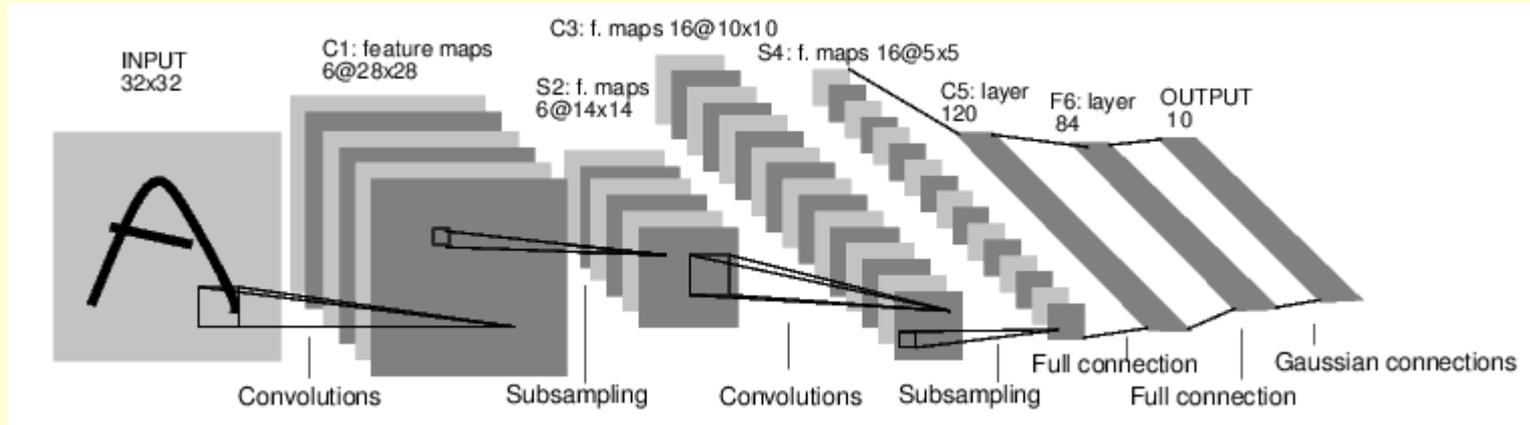


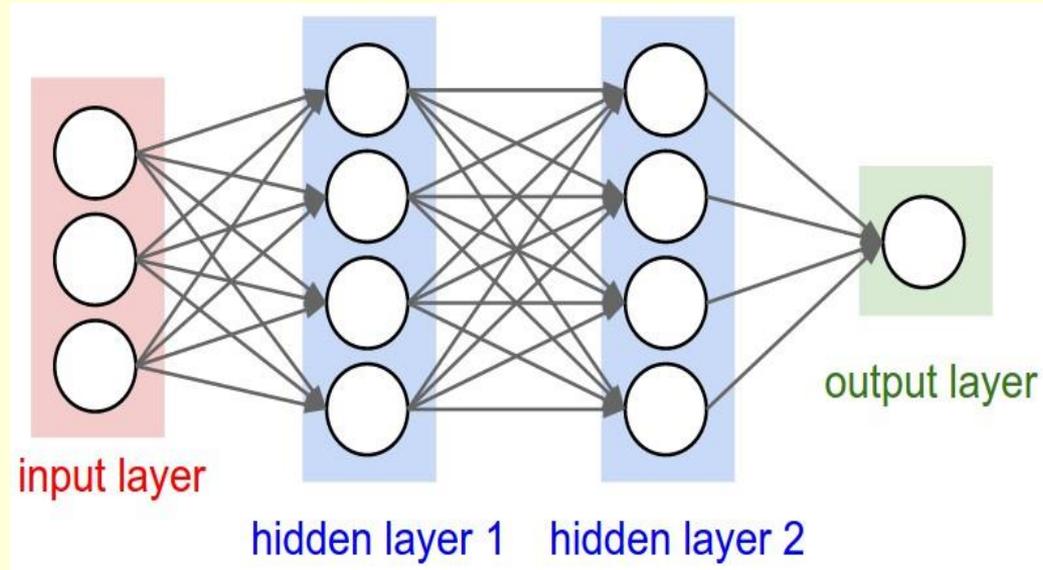
Convolutional Neural Networks (aka ConvNets, CNNs)



[LeNet-5, LeCun 1998]

Alyosha Efros (guest lecturer)

Neural Networks for Visual Data

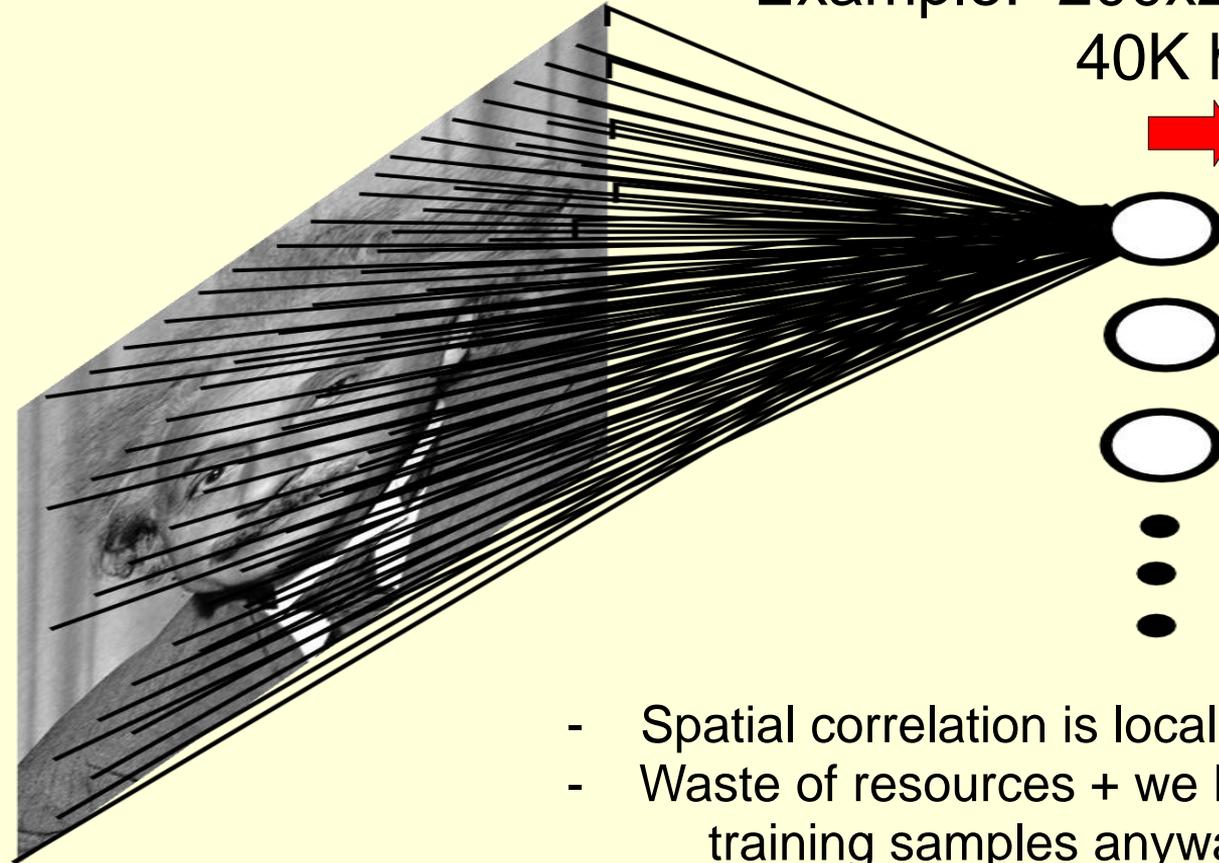


Fully Connected Layer

Example: 200x200 image

40K hidden units

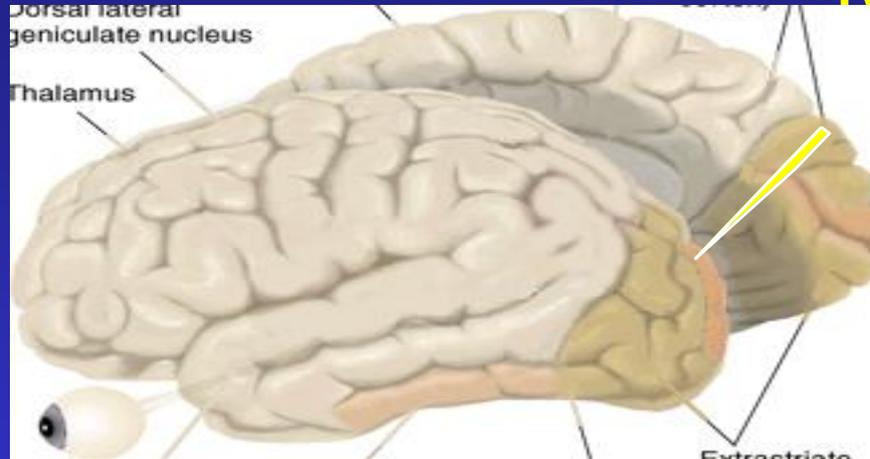
➔ ~2B parameters!!!



- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Single Cell Recording

Microelectrode

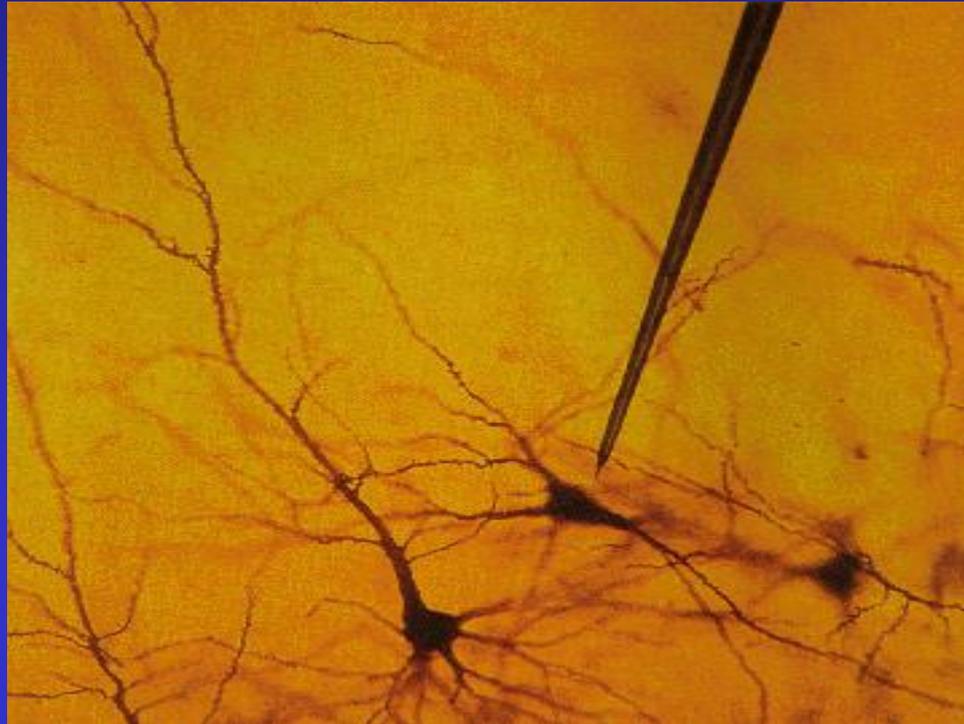


Amplifier

Electrical response
(action potentials)

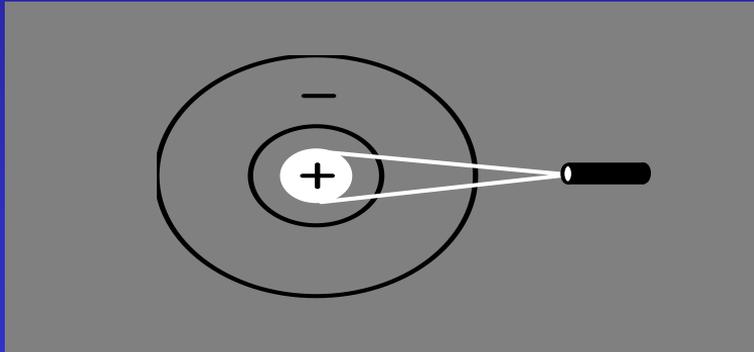


Single Cell Recording

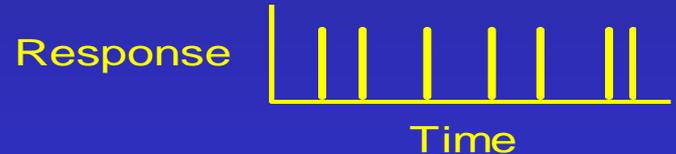


Retinal Receptive Fields

Receptive field structure in ganglion cells:
On-center Off-surround



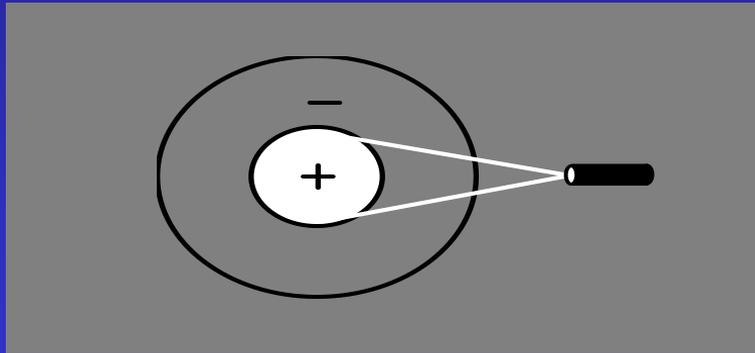
Stimulus condition



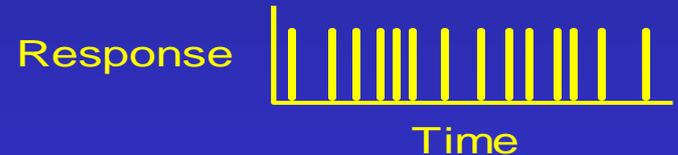
Electrical response

Retinal Receptive Fields

Receptive field structure in ganglion cells:
On-center Off-surround



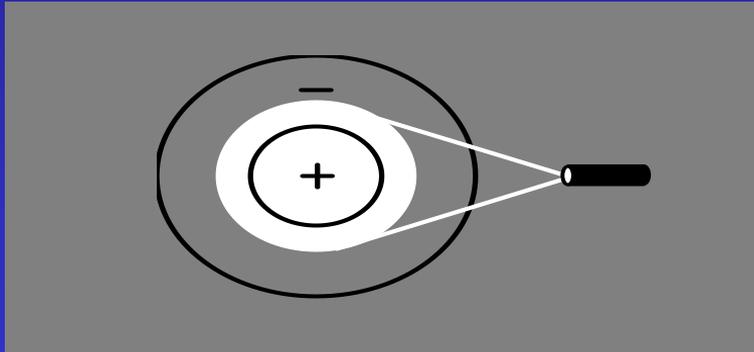
Stimulus condition



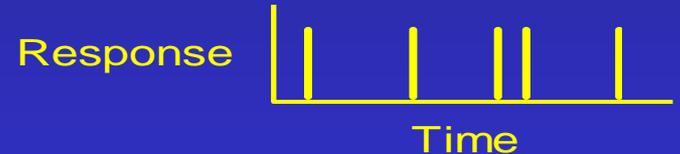
Electrical response

Retinal Receptive Fields

Receptive field structure in ganglion cells:
On-center Off-surround



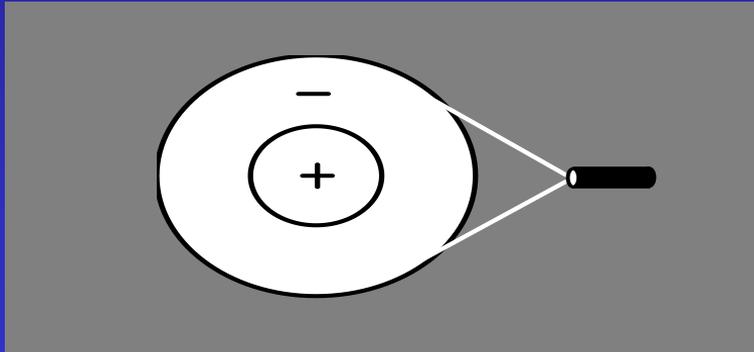
Stimulus condition



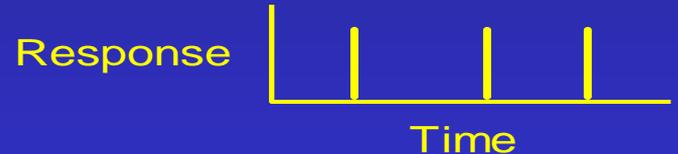
Electrical response

Retinal Receptive Fields

Receptive field structure in ganglion cells:
On-center Off-surround



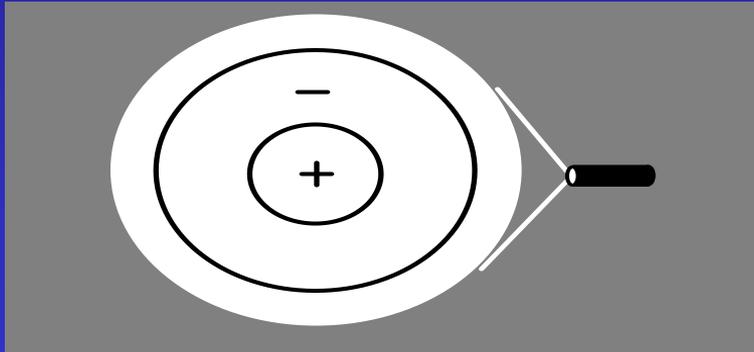
Stimulus condition



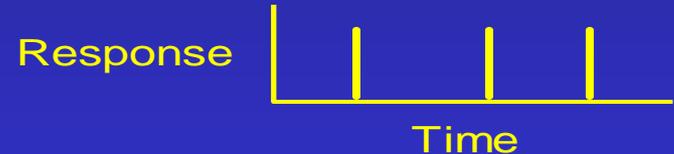
Electrical response

Retinal Receptive Fields

Receptive field structure in ganglion cells:
On-center Off-surround



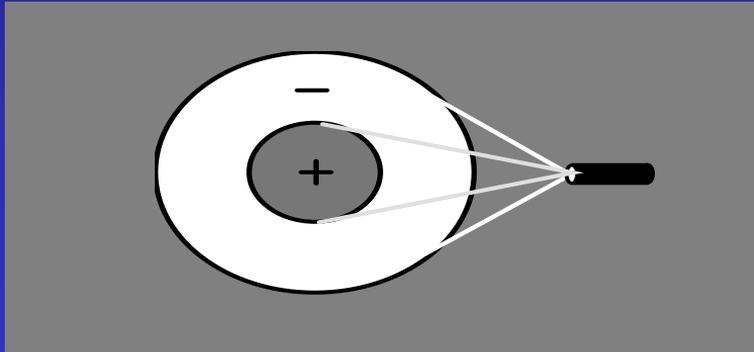
Stimulus condition



Electrical response

Retinal Receptive Fields

Receptive field structure in ganglion cells:
On-center Off-surround



Stimulus condition

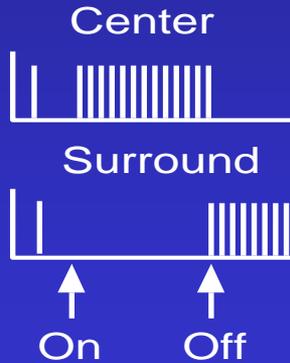


Electrical response

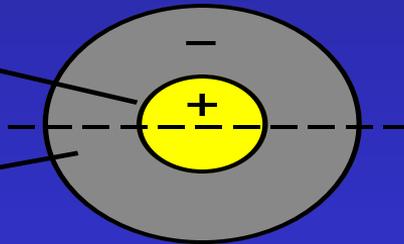
Retinal Receptive Fields

RF of On-center Off-surround cells

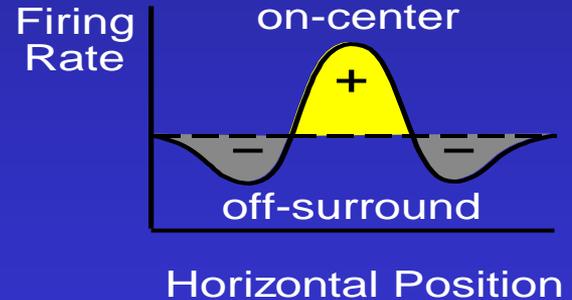
Neural Response



Receptive Field



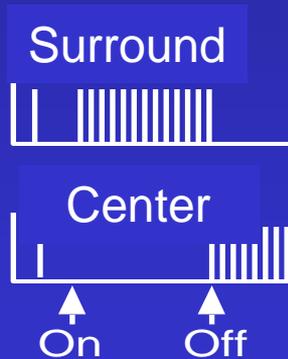
Response Profile



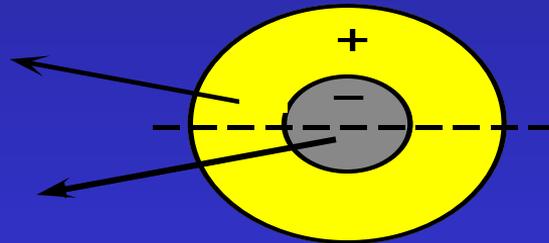
Retinal Receptive Fields

RF of Off-center On-surround cells

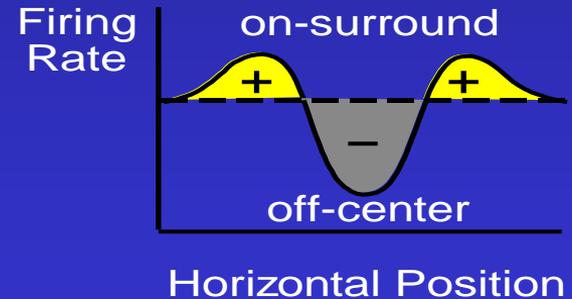
Neural Response



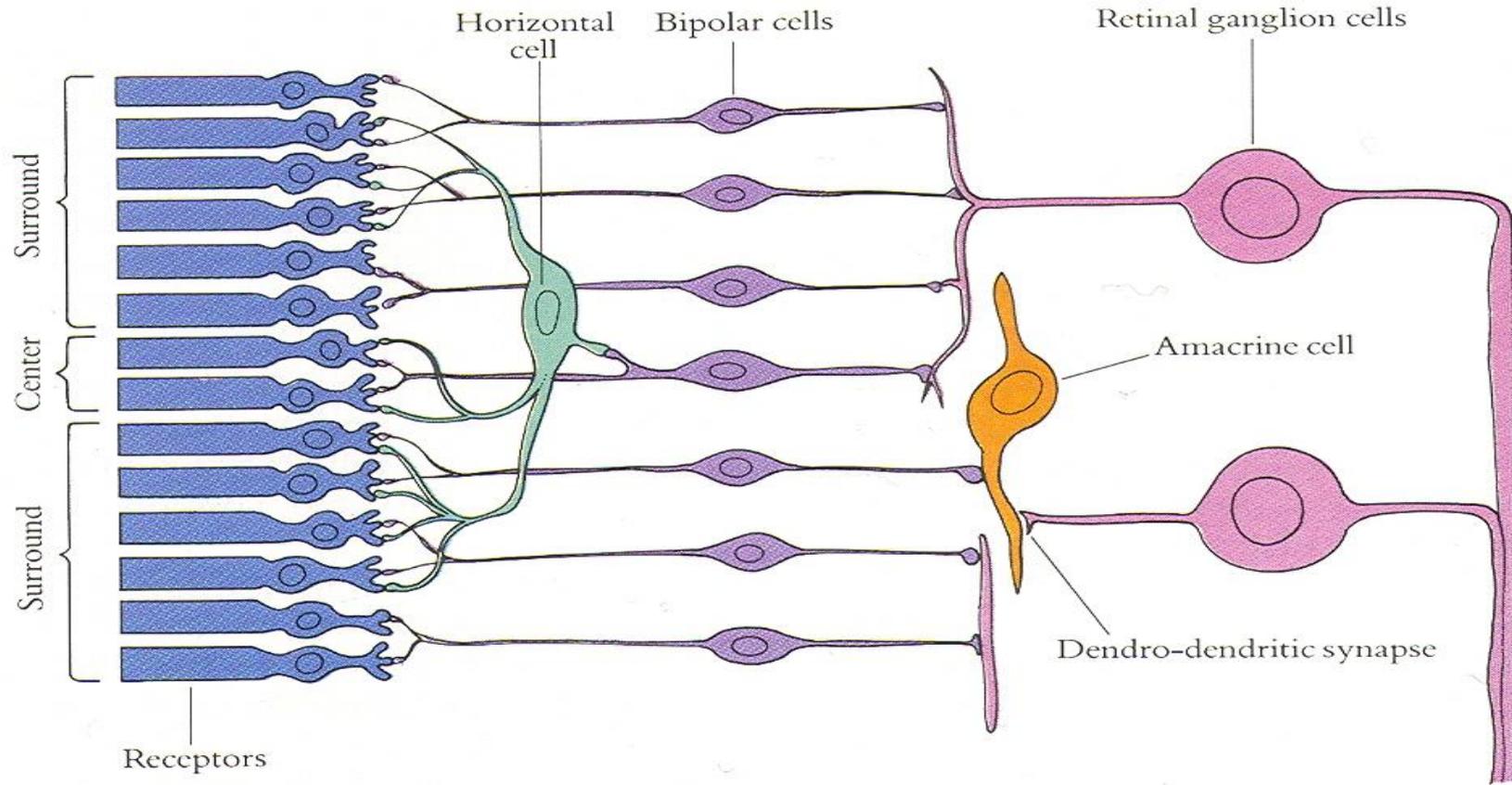
Receptive Field



Response Profile



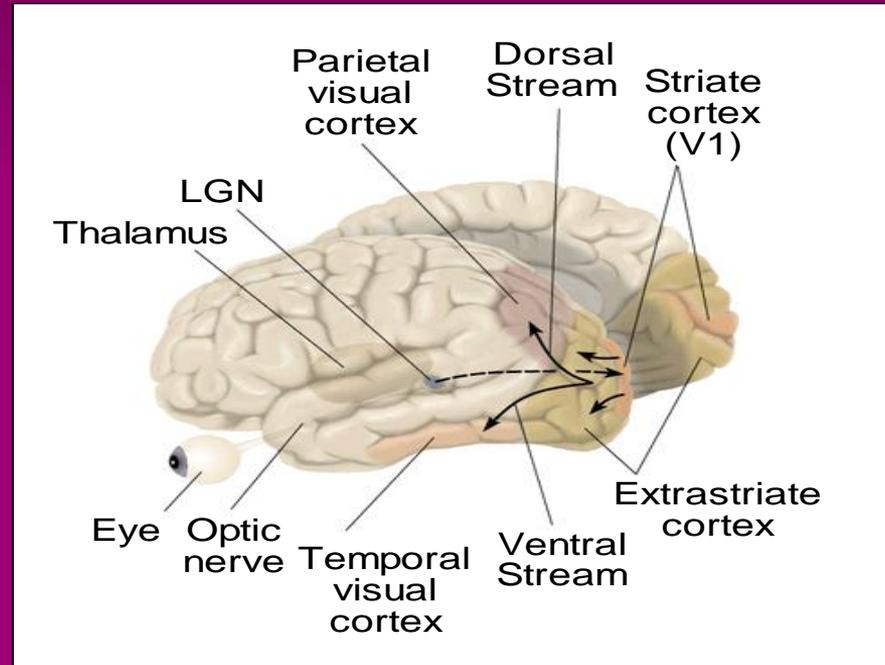
Retinal Receptive Fields



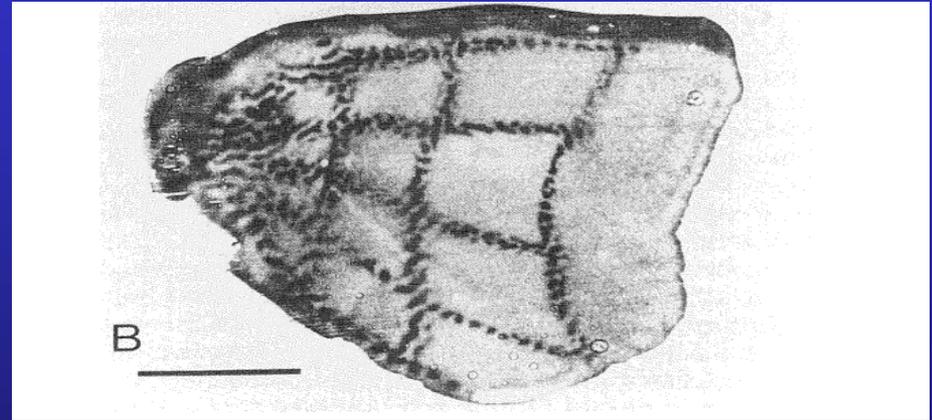
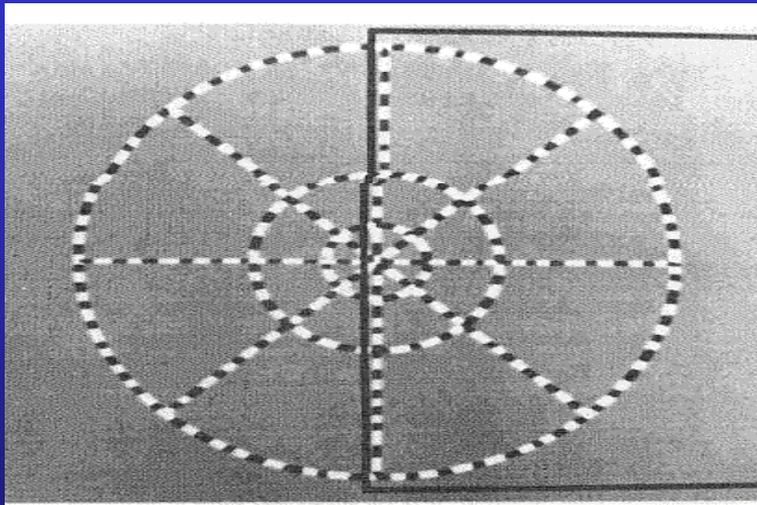
Visual Cortex

Cortical Area V1

aka:
Primary visual cortex
Striate cortex
Brodmann's area 17



Mapping from Retina to V1



Cortical Receptive Fields

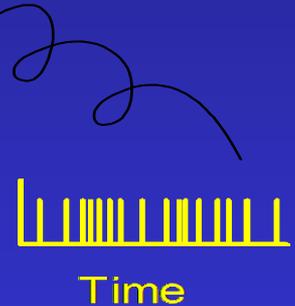
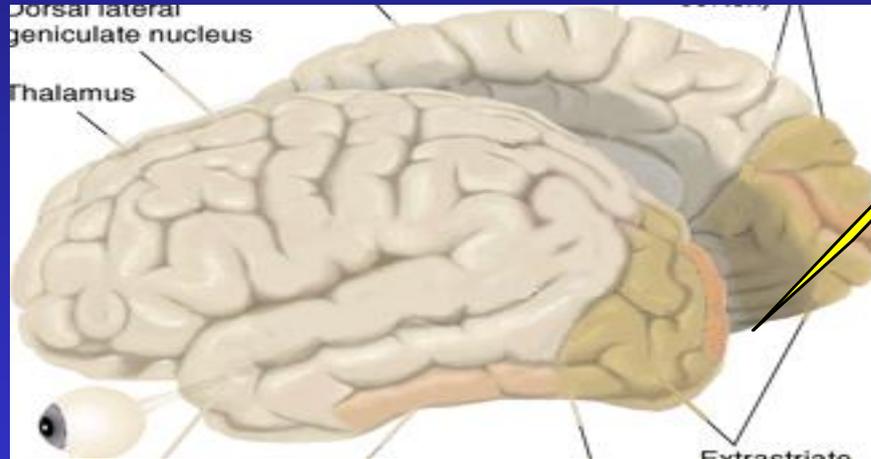
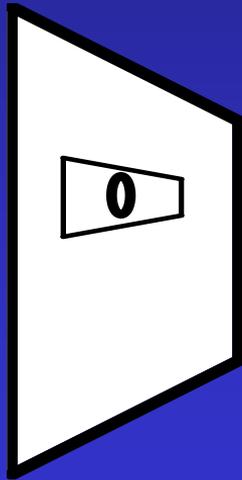
Single-cell recording from visual cortex



David Hubel & Thorston Wiesel

Cortical Receptive Fields

Single-cell recording from visual cortex



<https://www.youtube.com/watch?v=IOHayh06LJ4>

Cortical Receptive Fields

Three classes of cells in V1

Simple cells

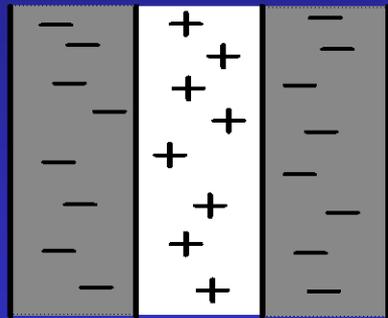
Complex cells

Hypercomplex cells

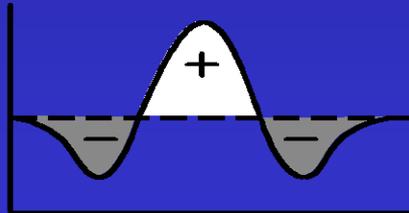
Cortical Receptive Fields

Simple Cells: "Line Detectors"

A. Light Line Detector

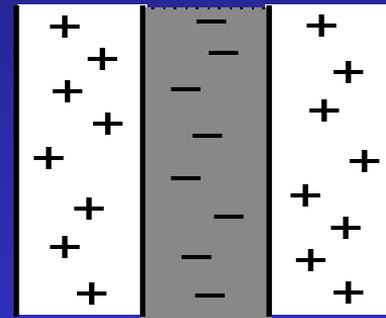


Firing Rate

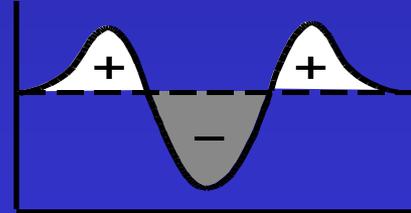


Horizontal Position

B. Dark Line Detector



Firing Rate

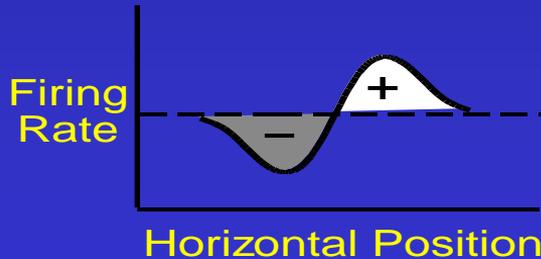
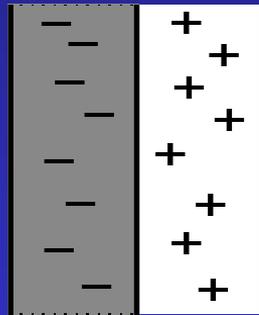


Horizontal Position

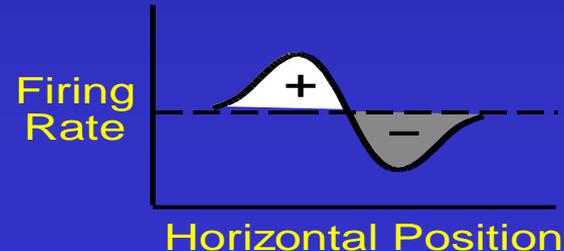
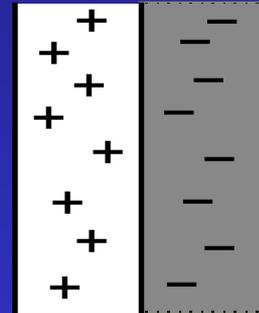
Cortical Receptive Fields

Simple Cells: "Edge Detectors"

C. Dark-to-light Edge Detector

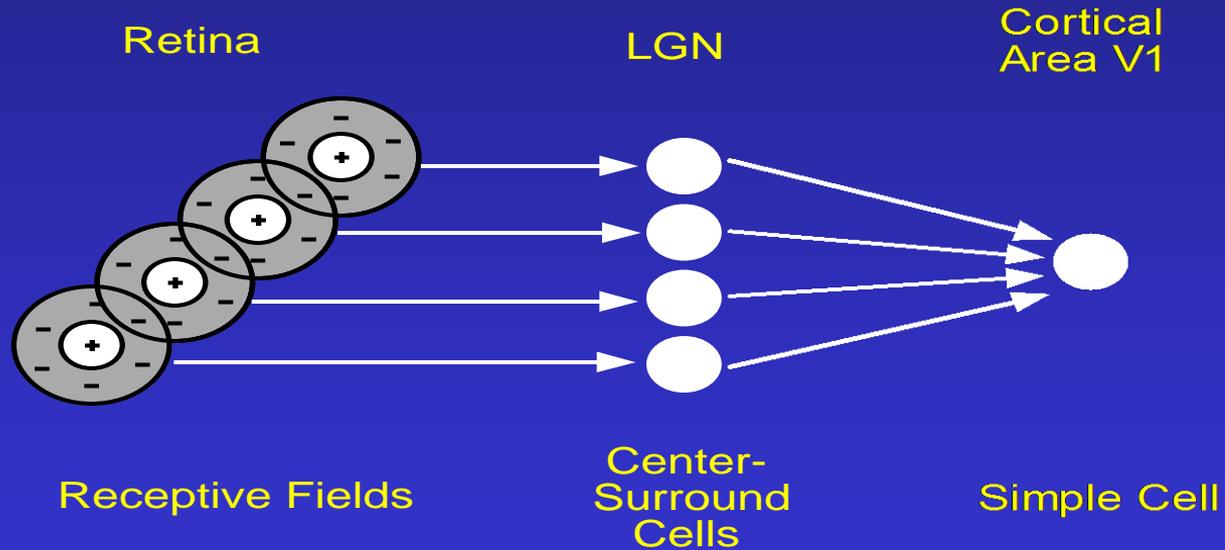


D. Light-to-dark Edge Detector



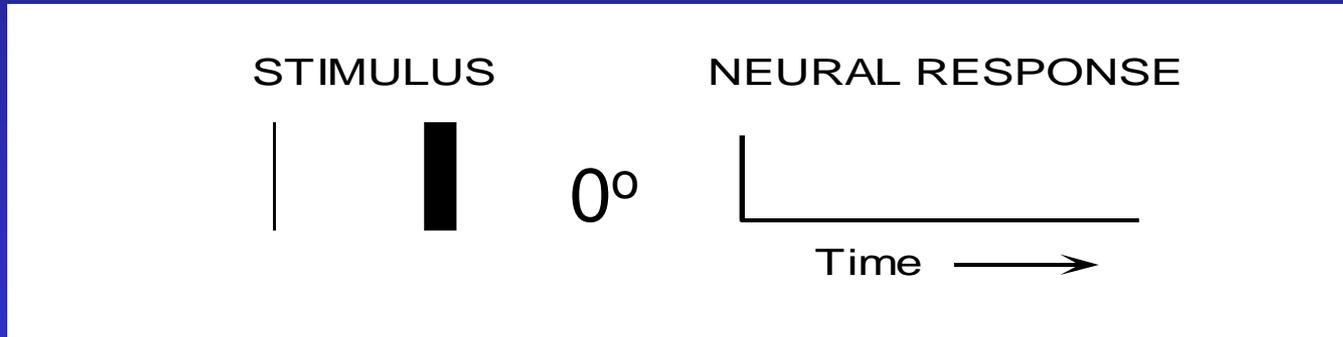
Cortical Receptive Fields

Constructing a line detector



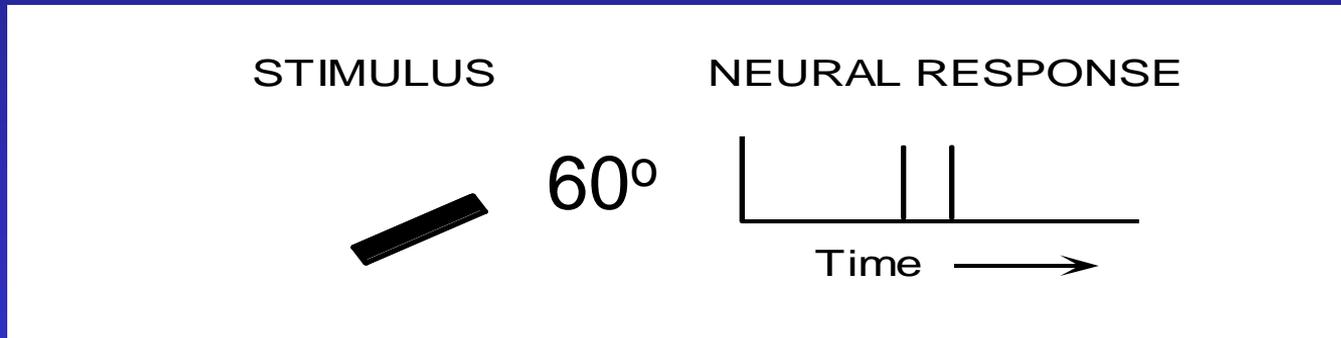
Cortical Receptive Fields

Complex Cells



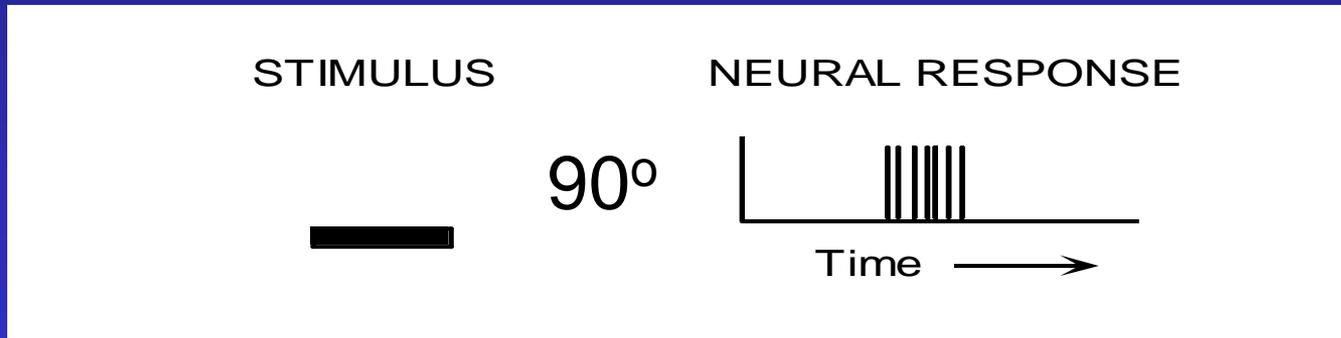
Cortical Receptive Fields

Complex Cells



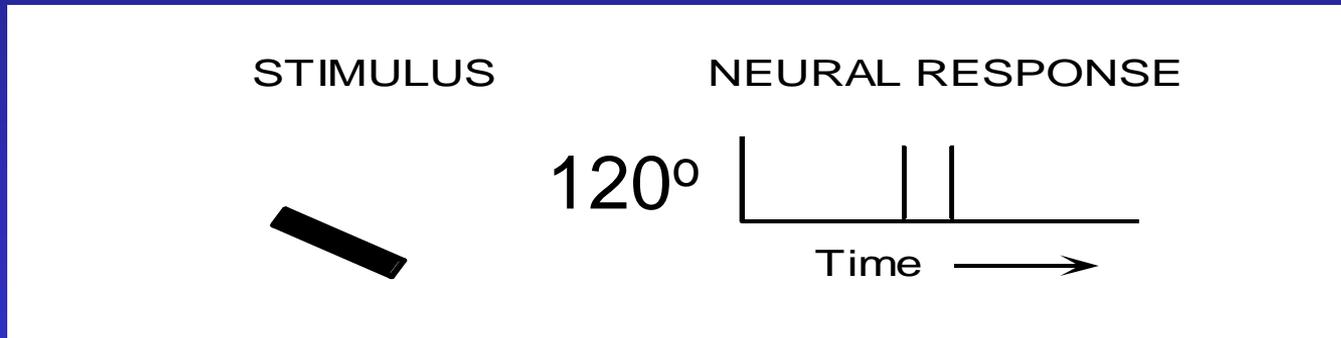
Cortical Receptive Fields

Complex Cells



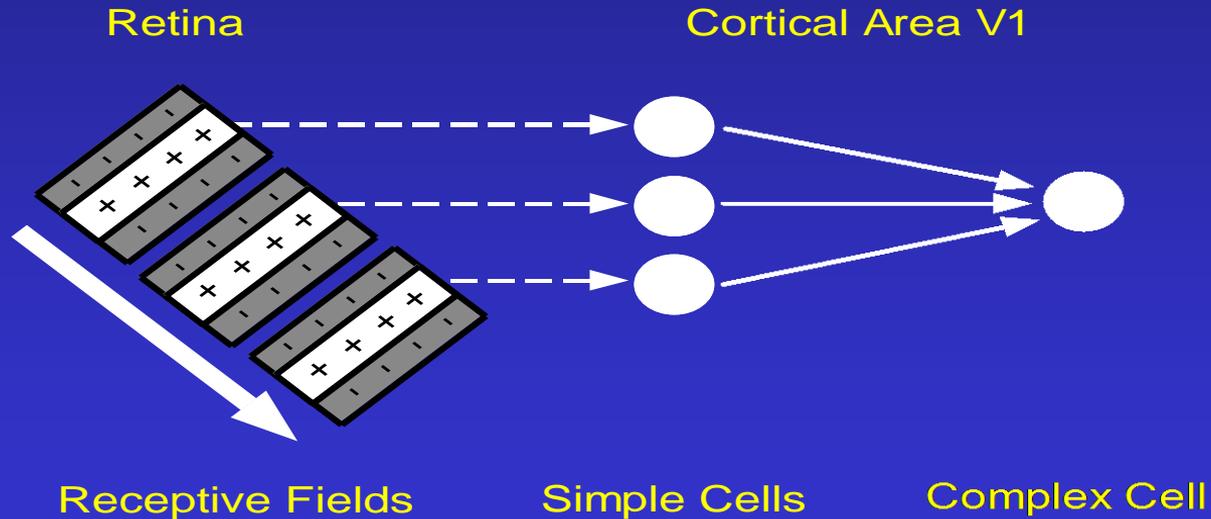
Cortical Receptive Fields

Complex Cells



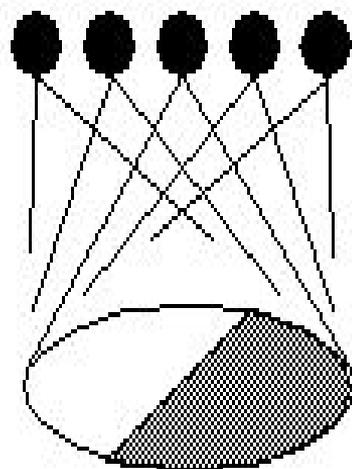
Cortical Receptive Fields

Constructing a Complex Cell



Hubel & Weisel

topographical mapping

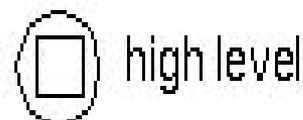
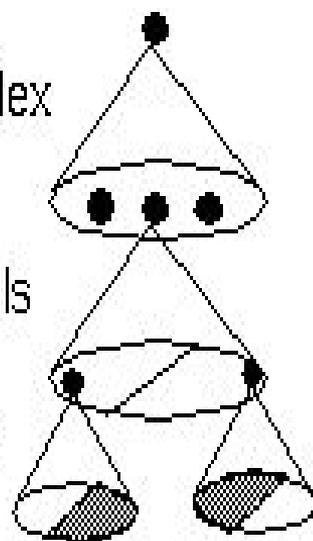


featural hierarchy

hyper-complex cells

complex cells

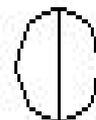
simple cells



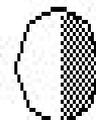
high level



mid level

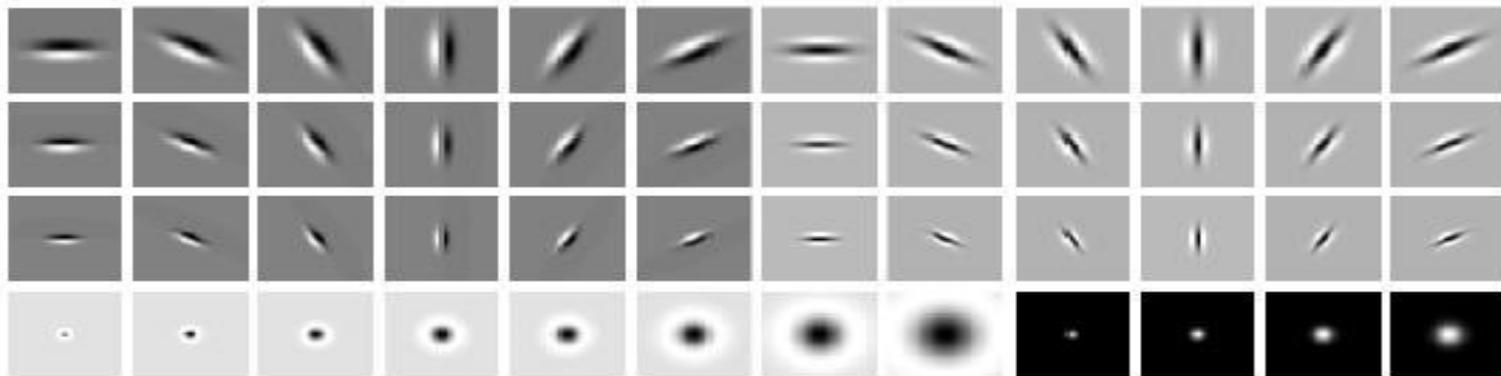


low level



Modeling V1 in Computer Vision

LM Filter Bank

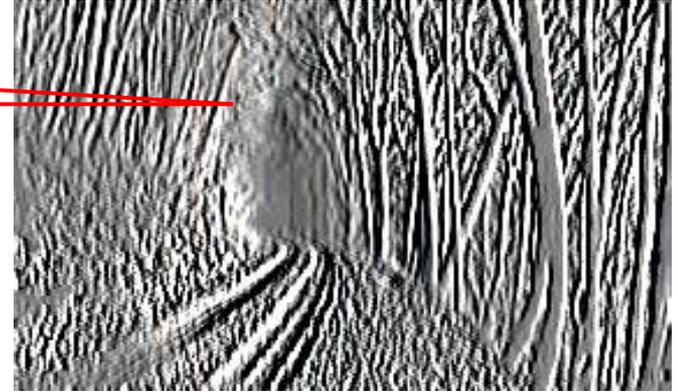


Code for filter banks: www.robots.ox.ac.uk/~vgg/research/texclass/filters.html

Convolutional of Two Signals



$$* \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} =$$



$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1,y-n_2]$$

elementwise multiplication and
sum of a filter and the signal
(image)

Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10							

Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20						

Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20	30					

Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20	30	30				

Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20	30	30				
							?		
				50					

Image filtering

$$h[\cdot, \cdot] = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

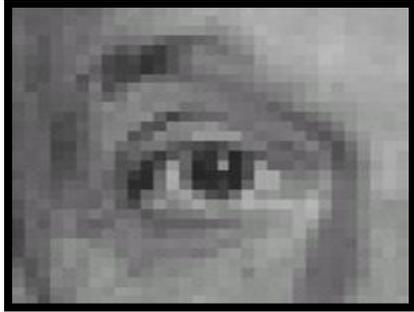
$$f[\cdot, \cdot]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$g[\cdot, \cdot]$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

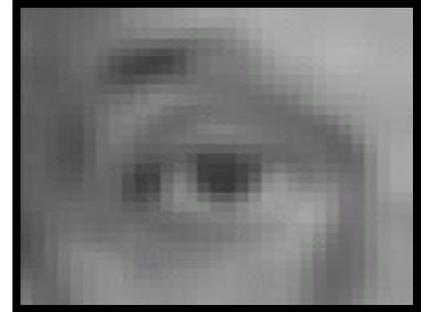
Linear filters: examples



Original

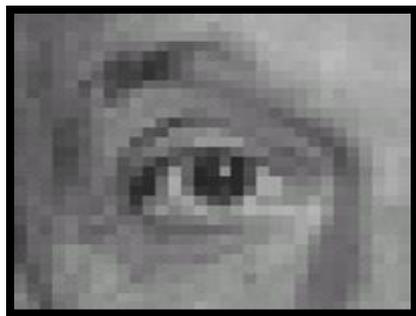
$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

=



Blur (with a mean filter)

Practice with linear filters



Original

0	0	0
0	1	0
0	0	0

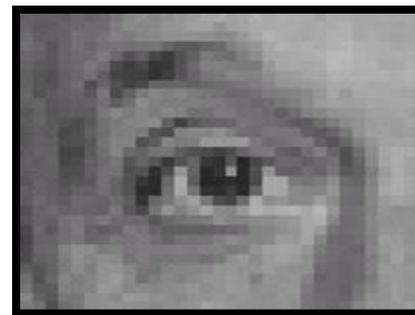
?

Practice with linear filters



Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)

Practice with linear filters



Original

0	0	0
0	0	1
0	0	0

?

Practice with linear filters

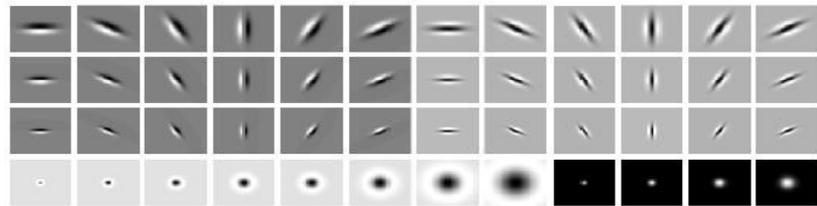


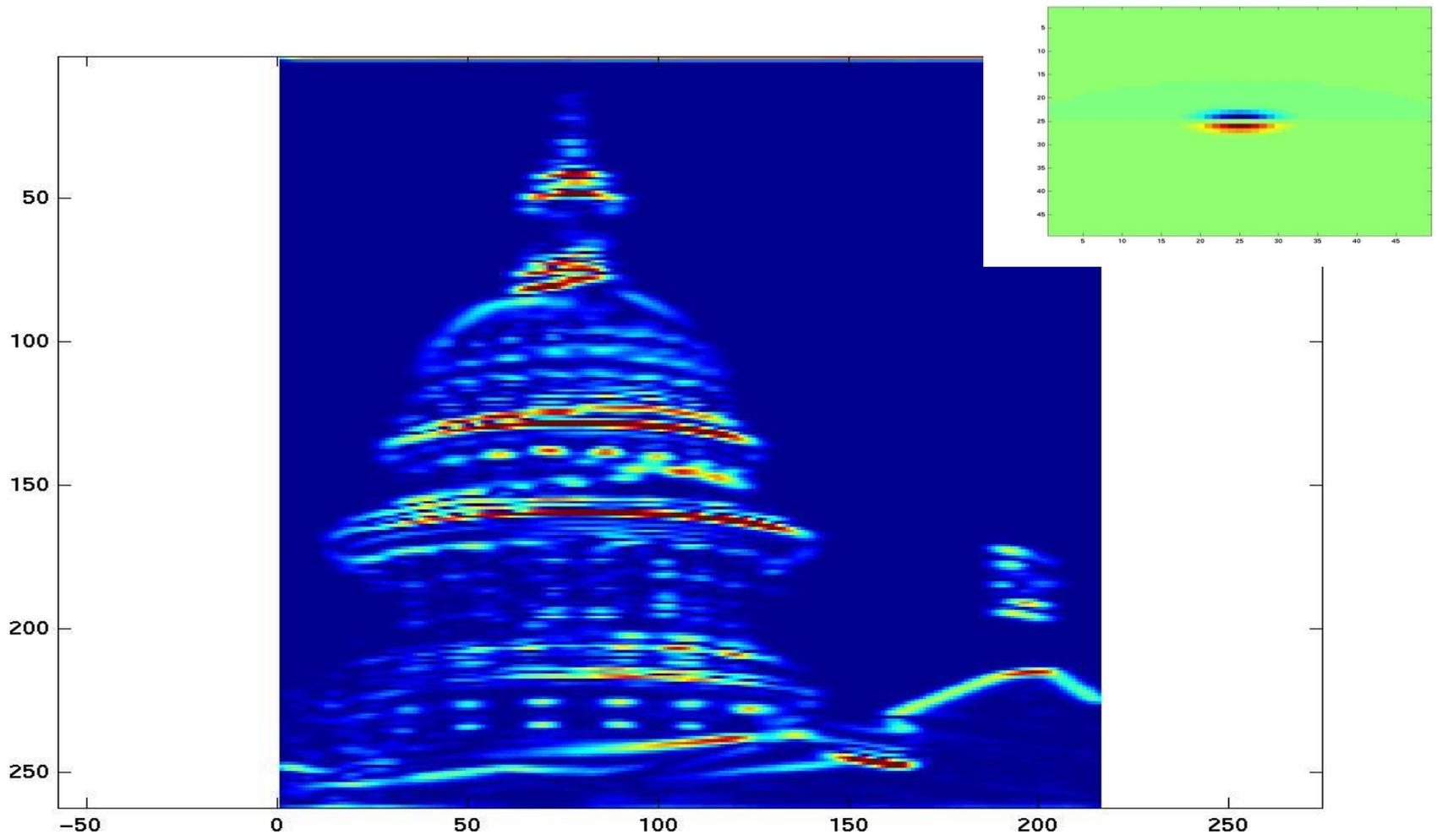
Original

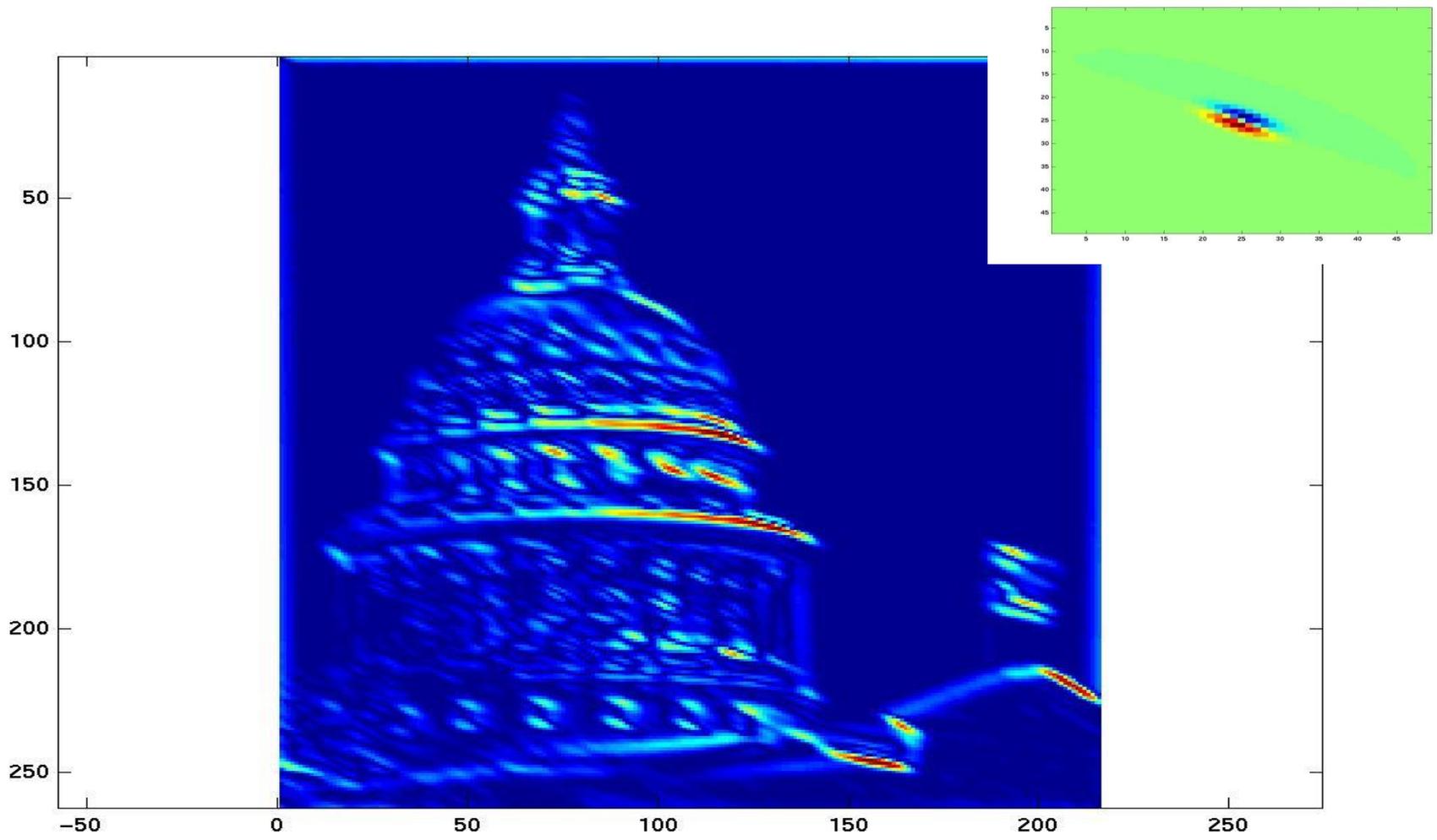
0	0	0
0	0	1
0	0	0

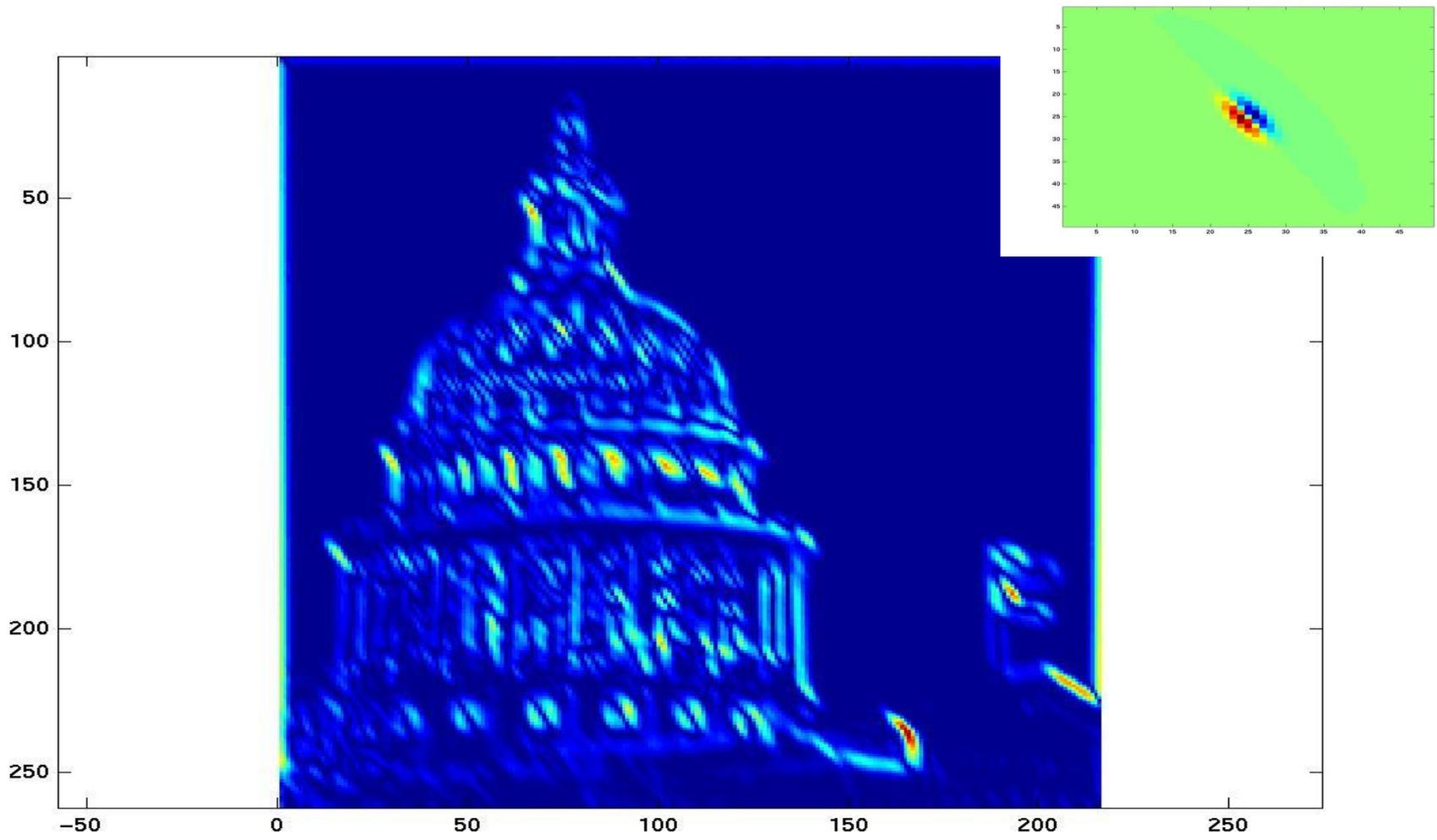


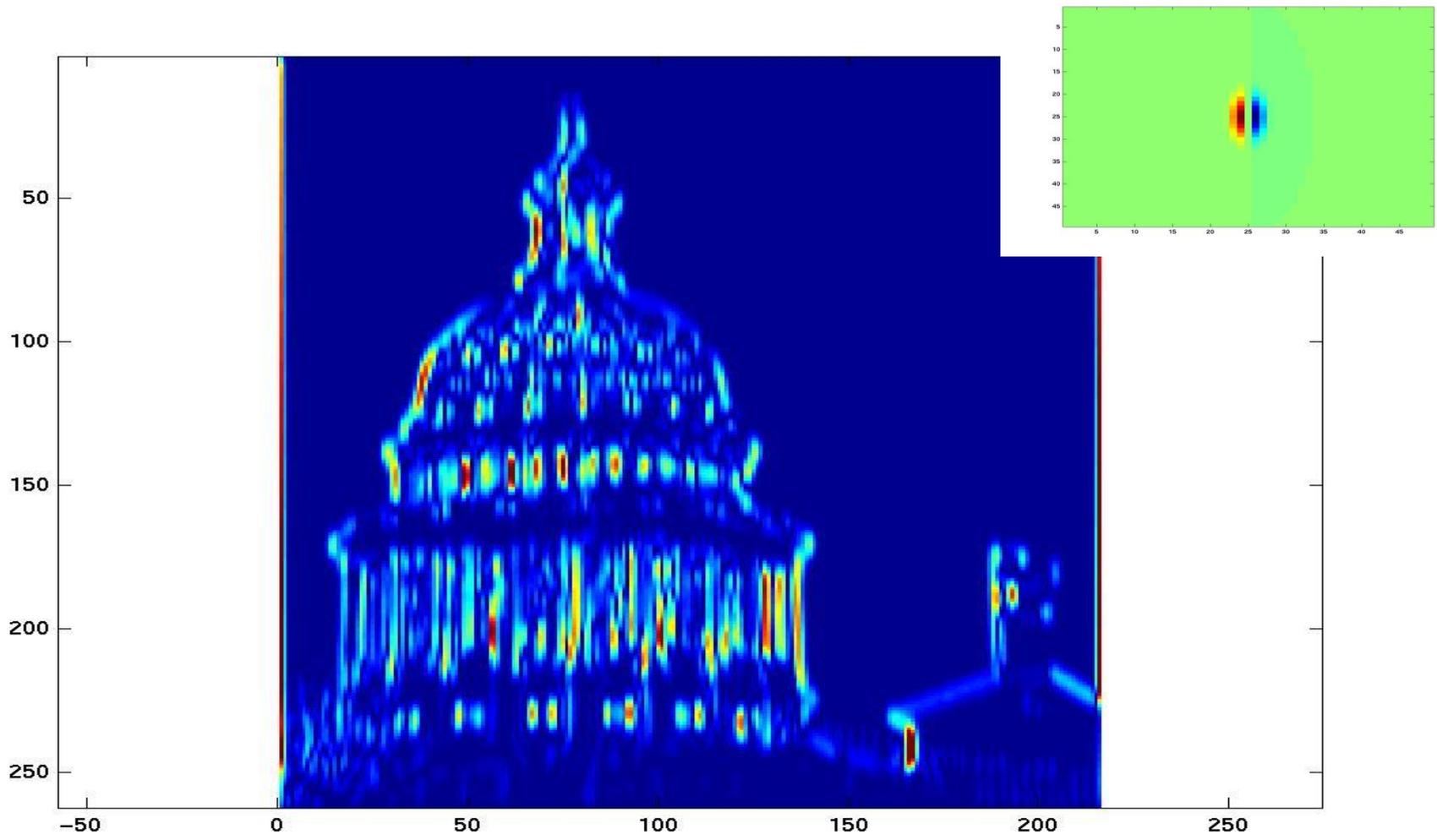
Shifted left
By 1 pixel

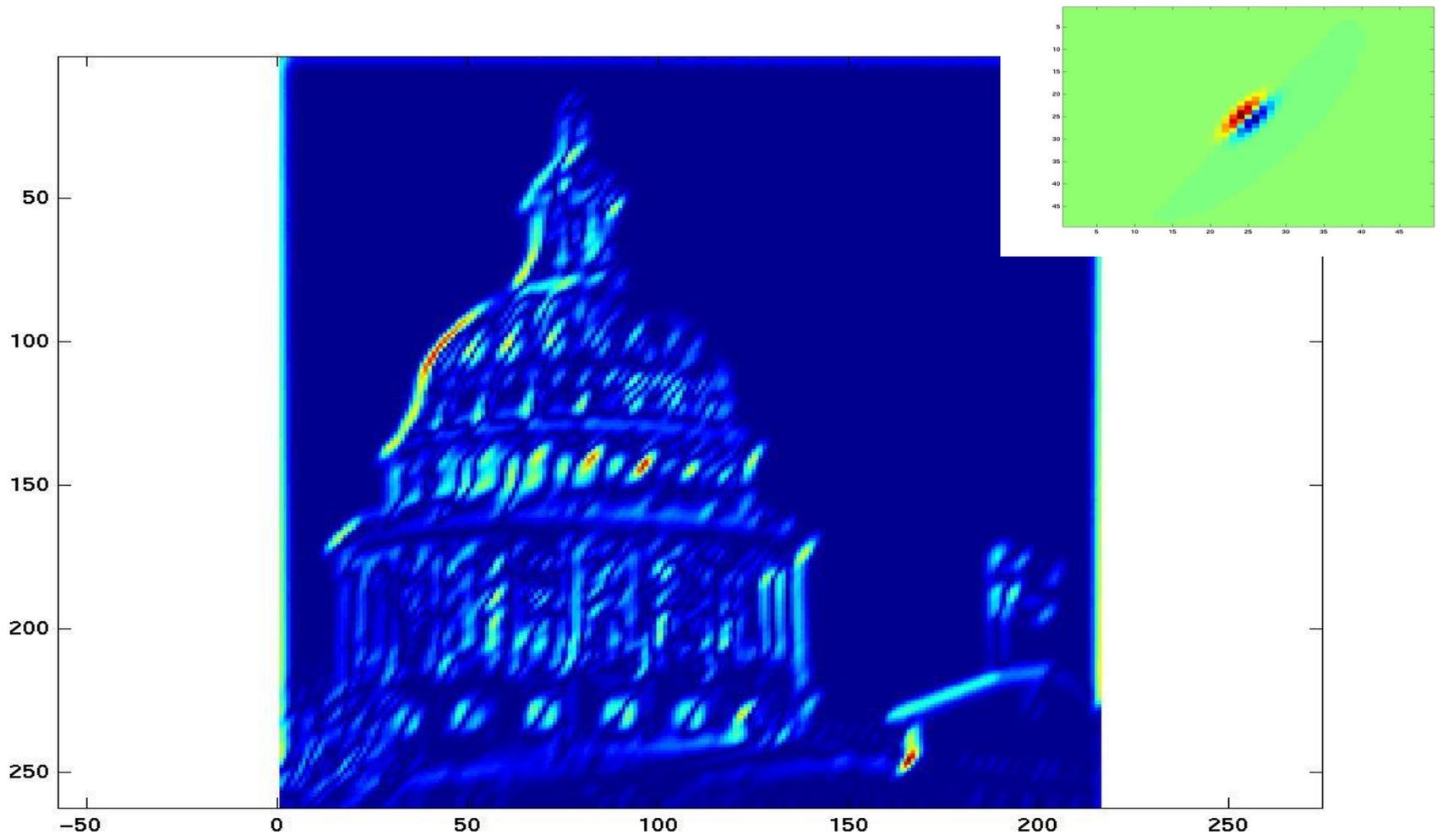


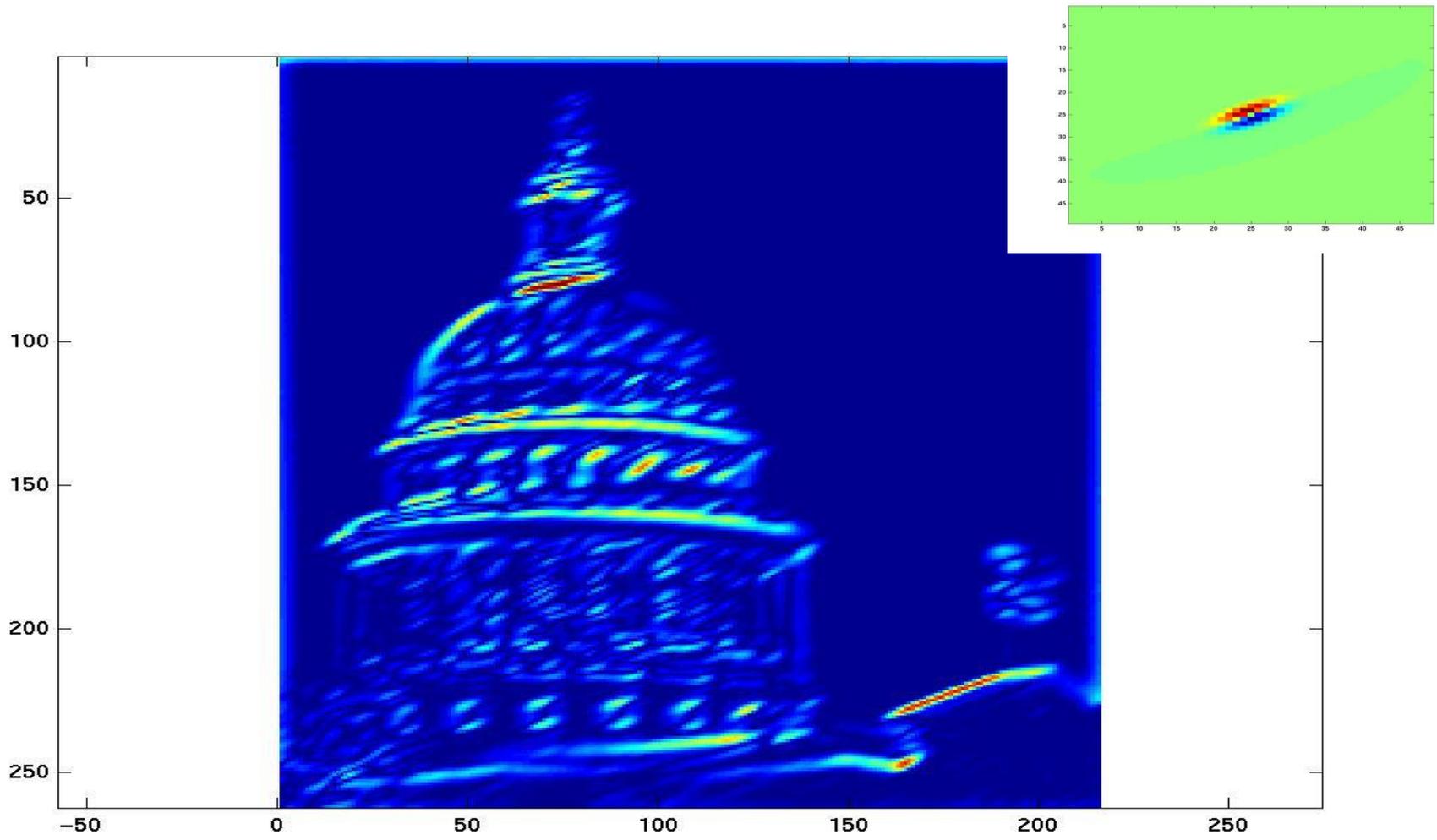


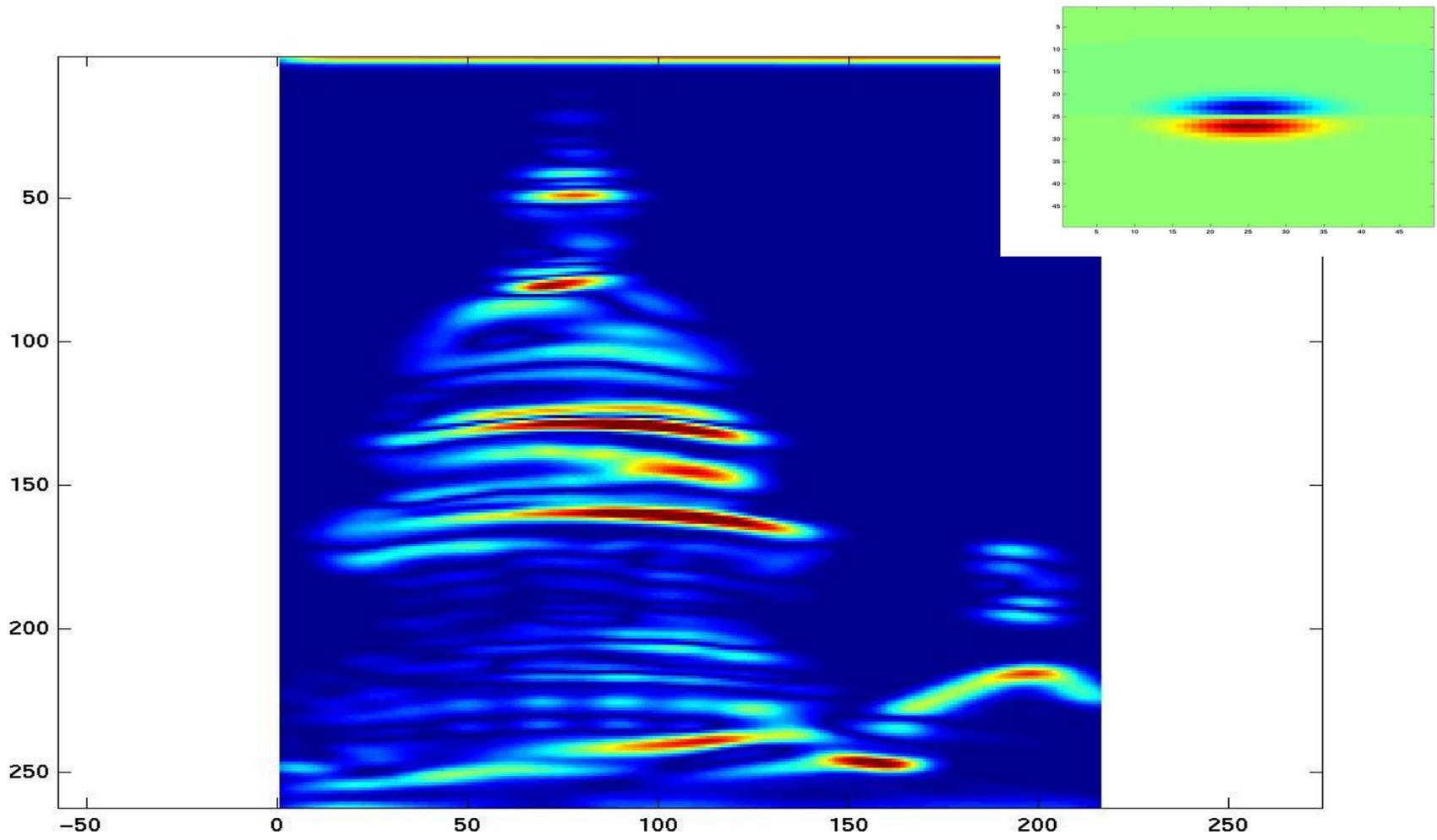


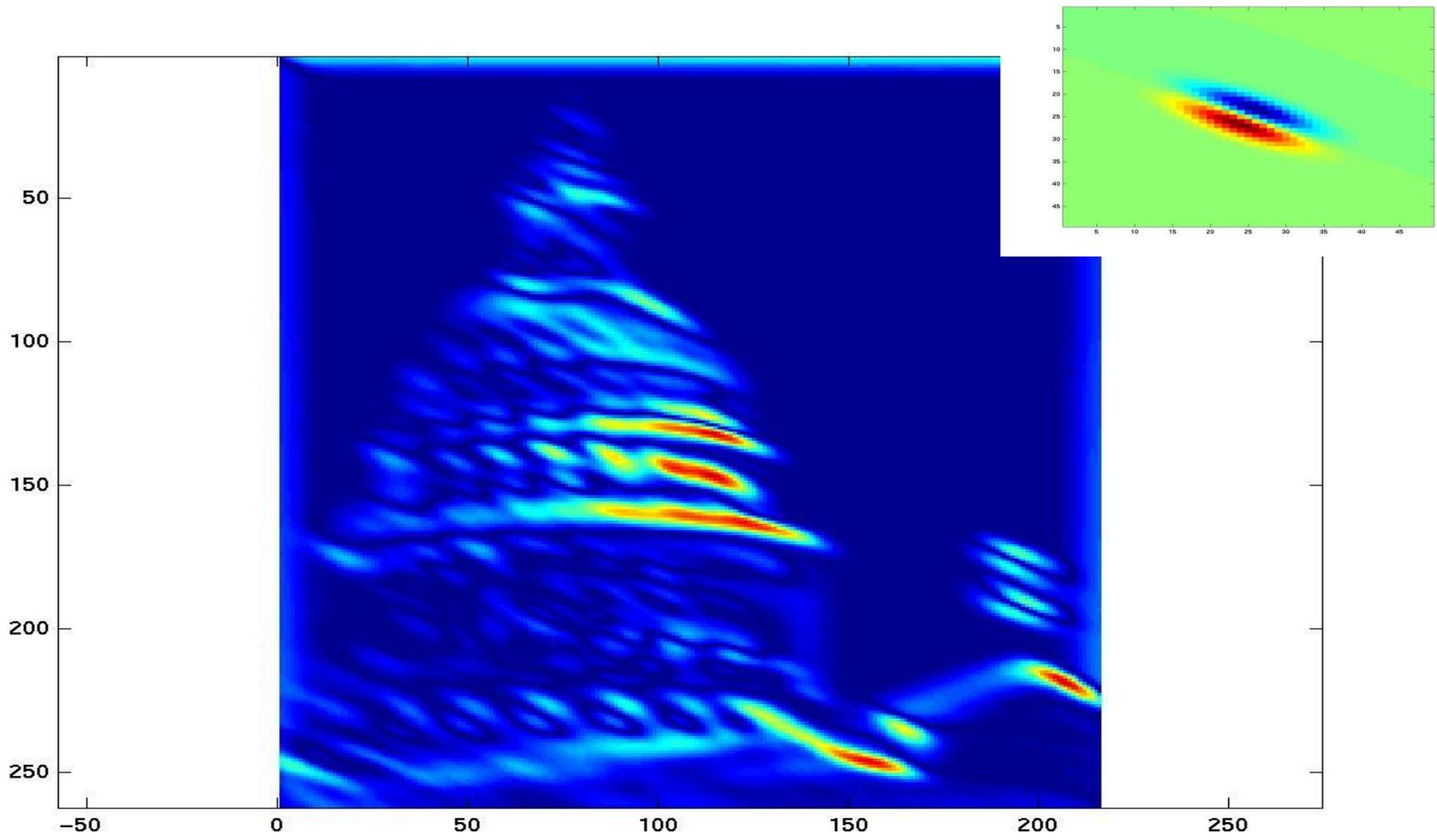


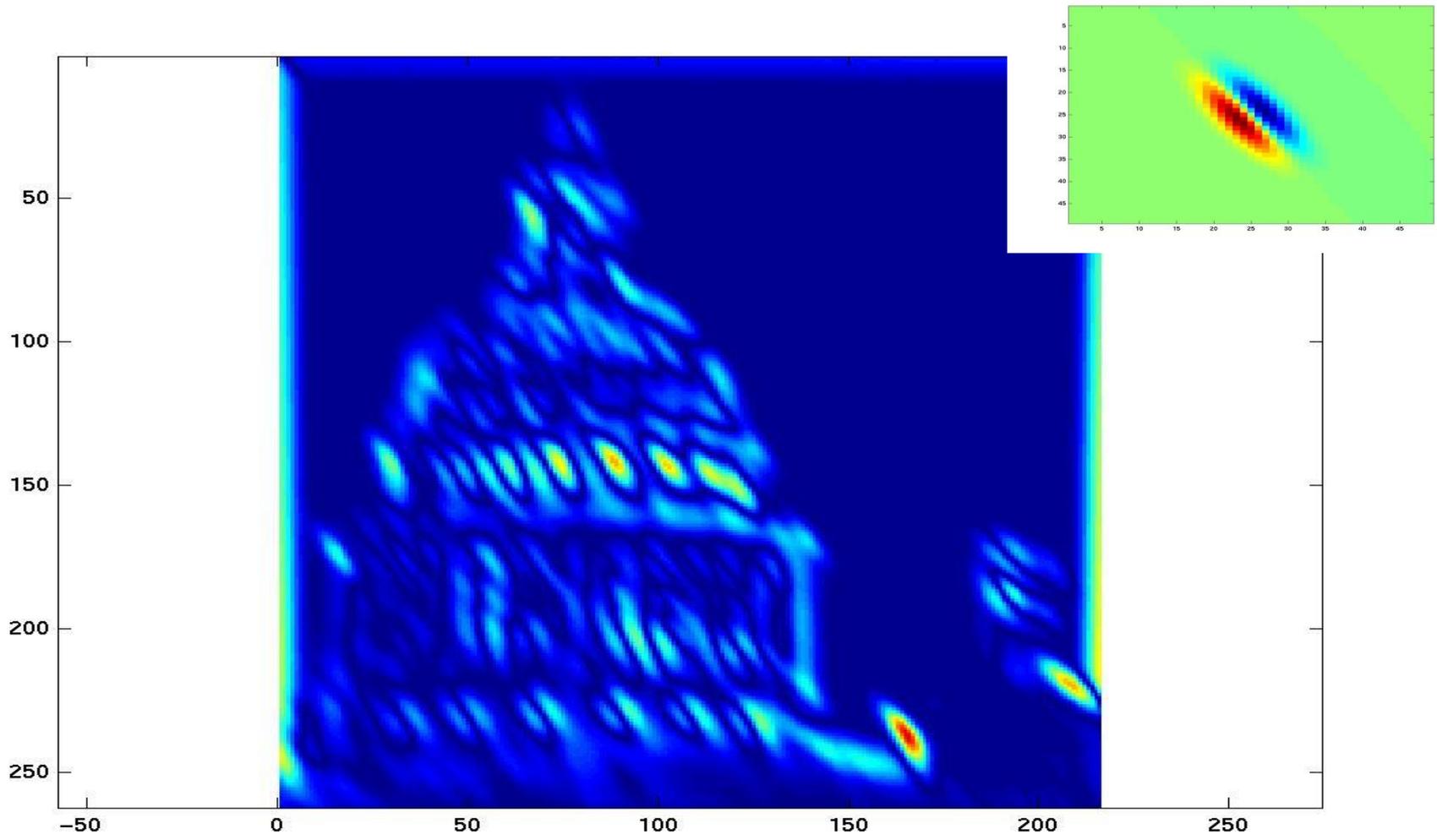


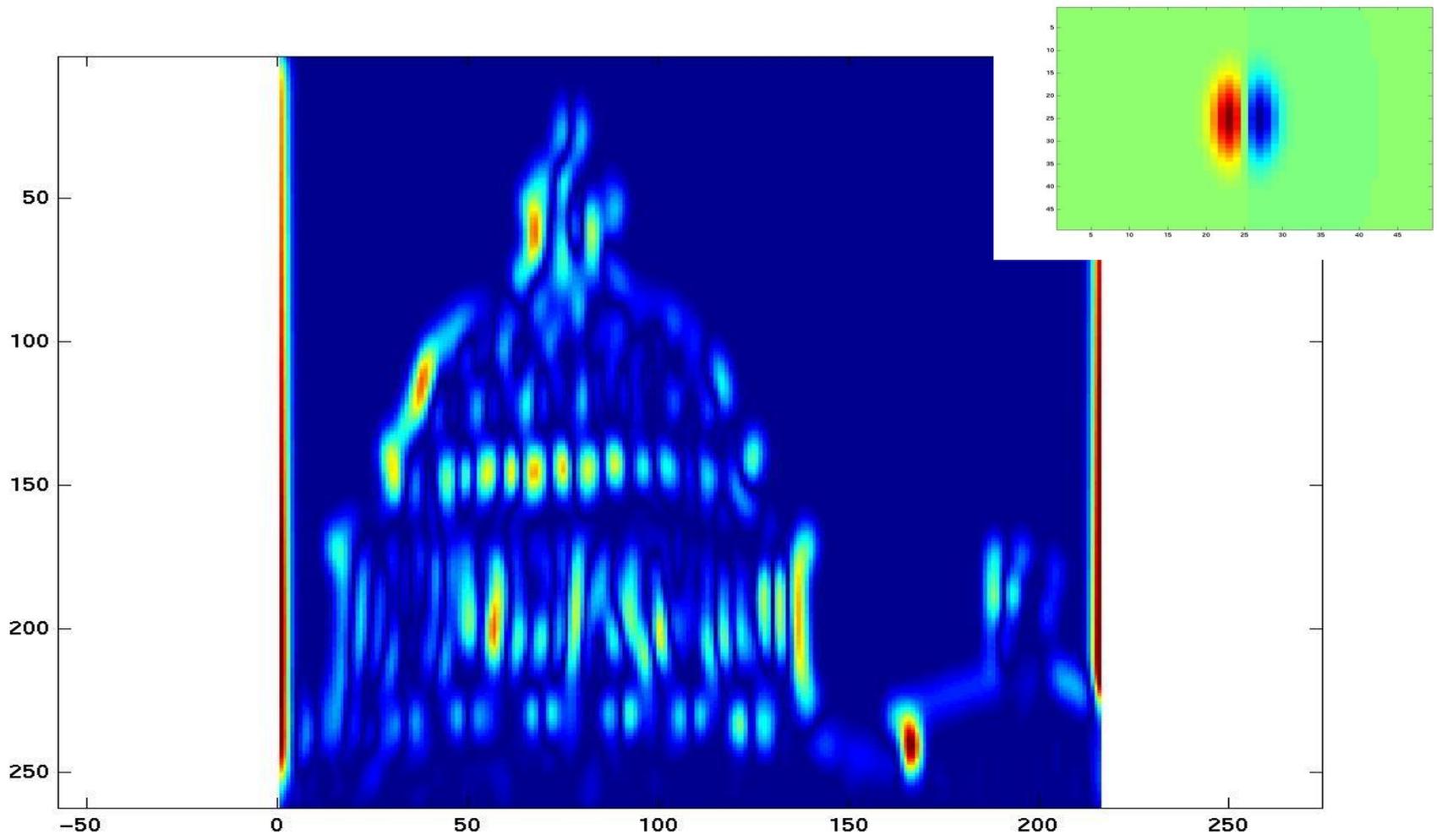


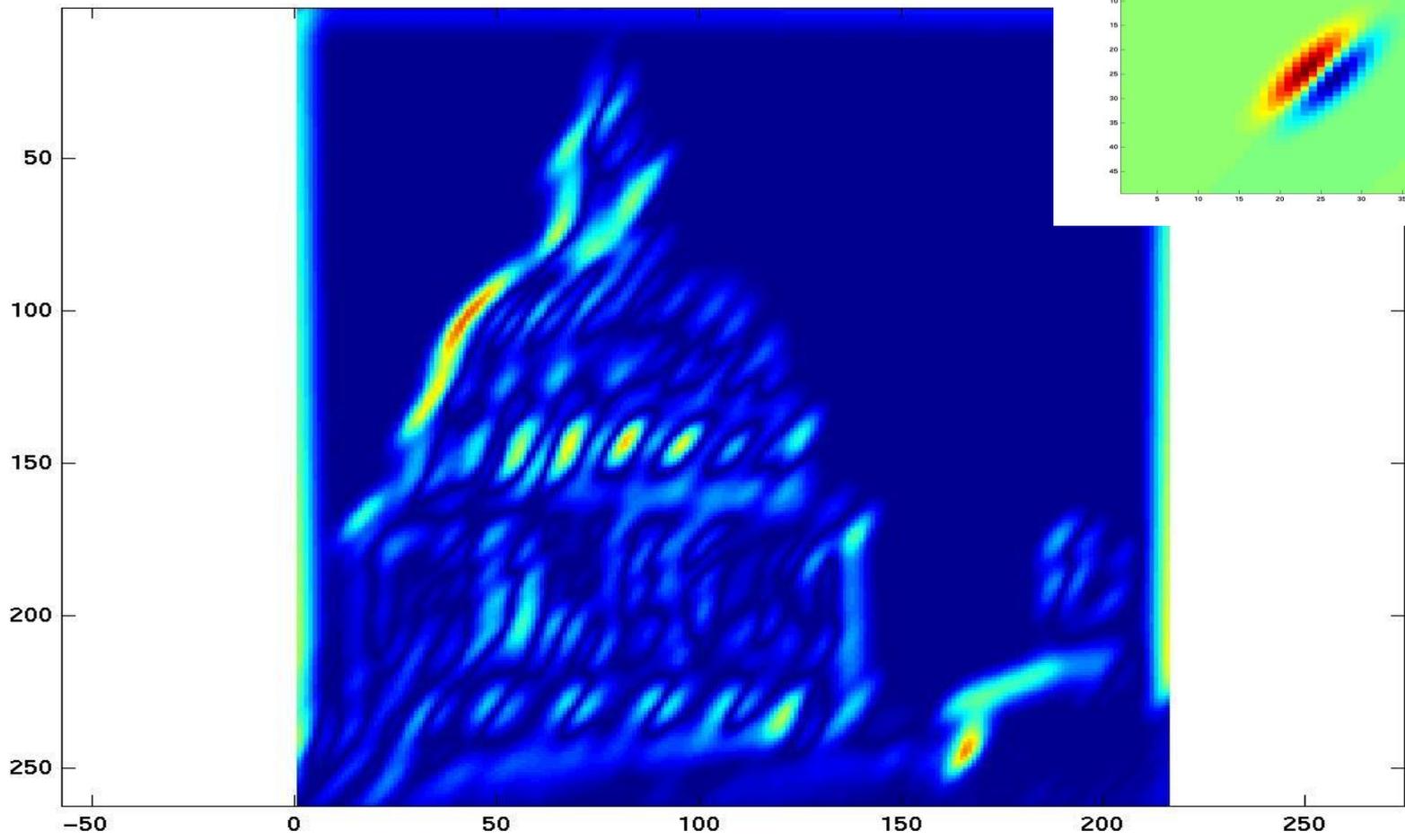


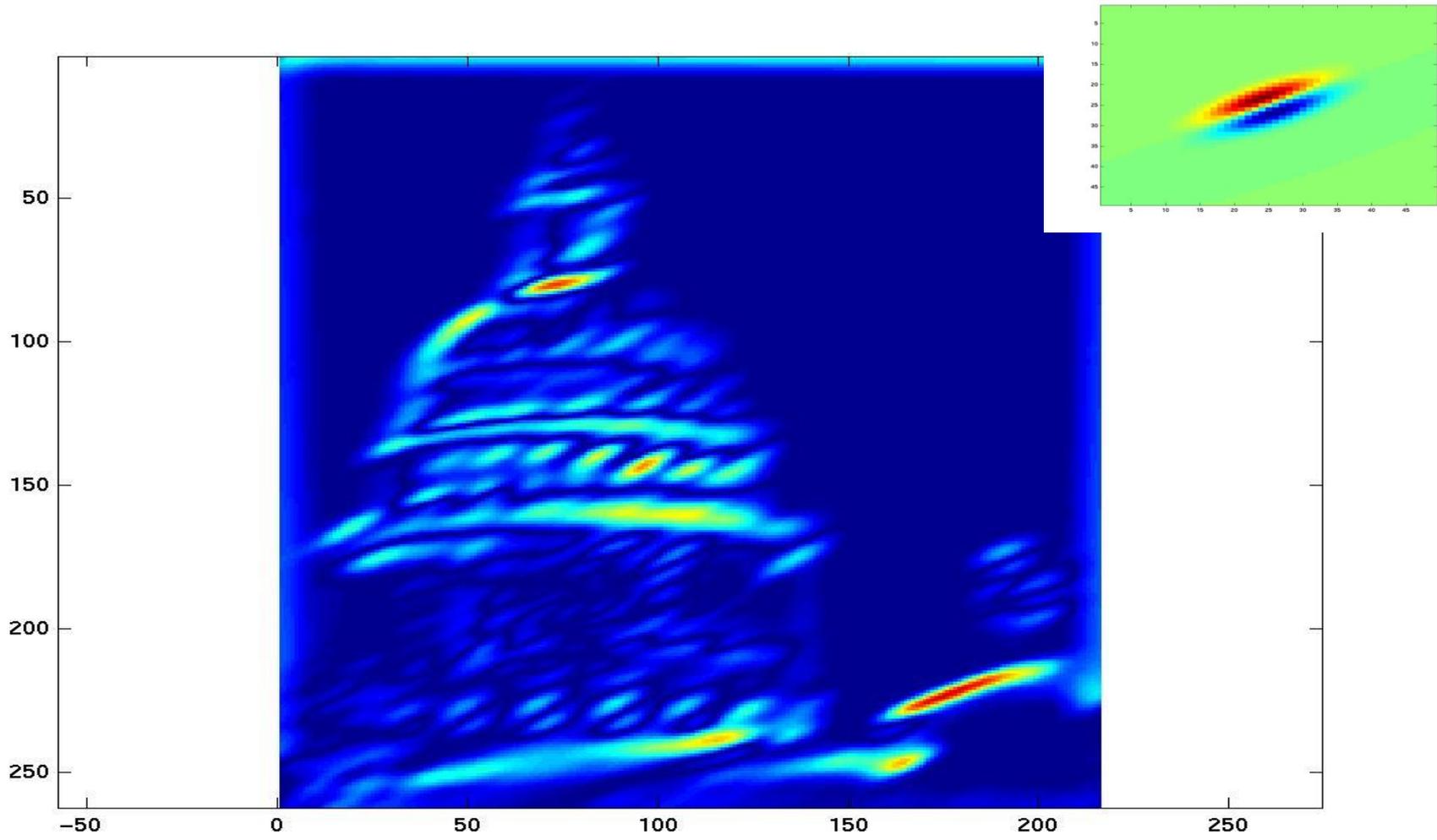


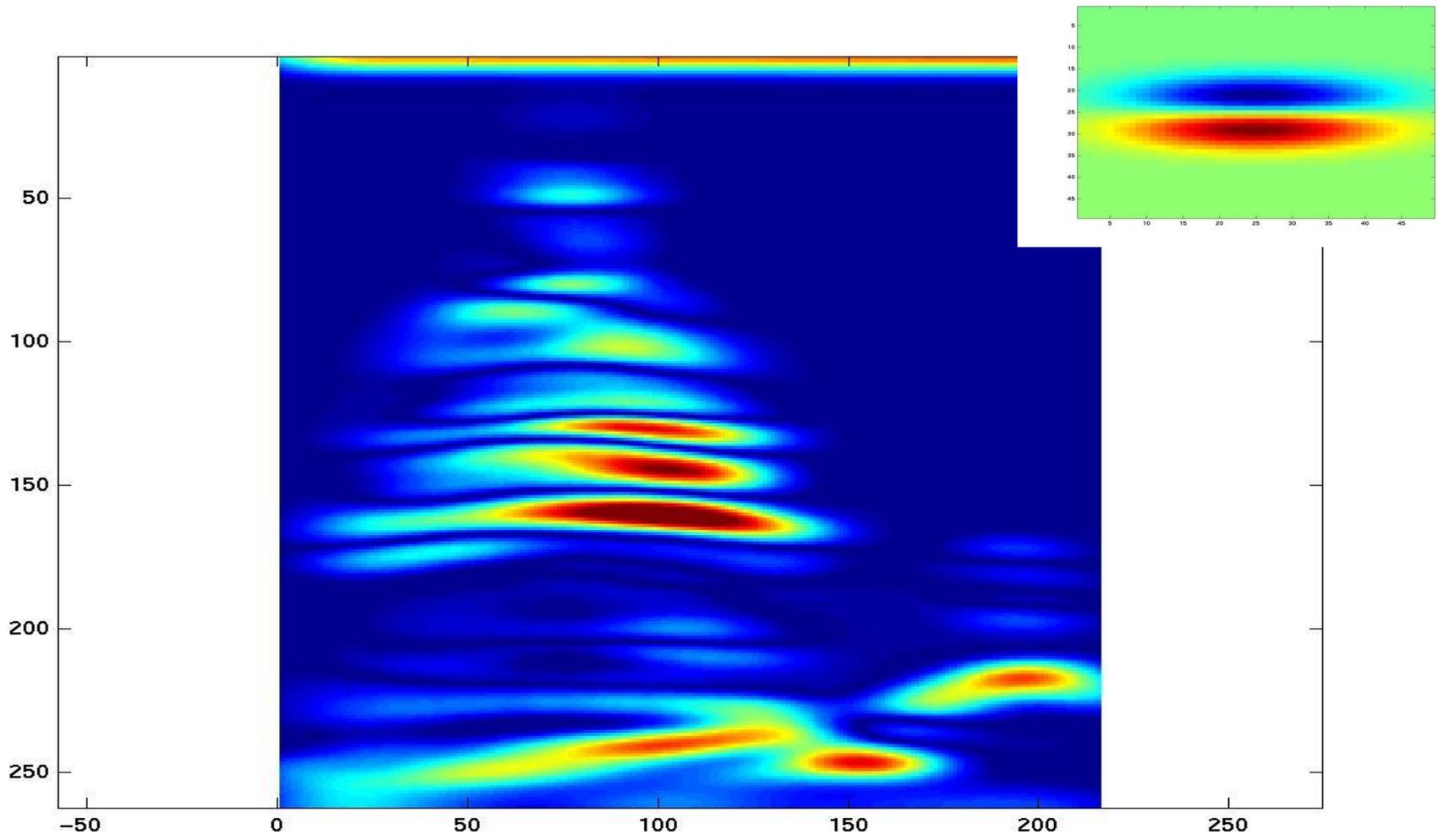


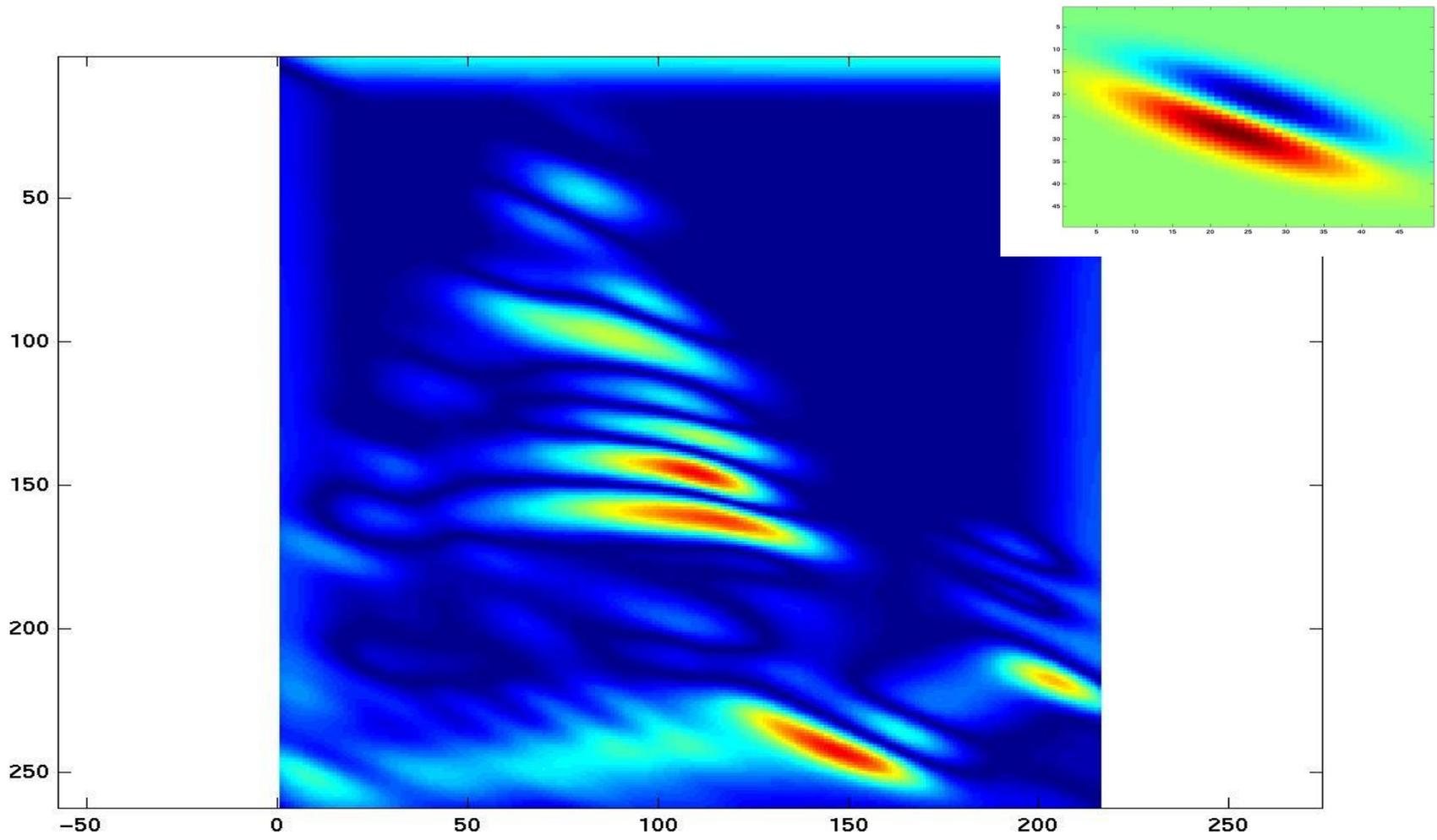


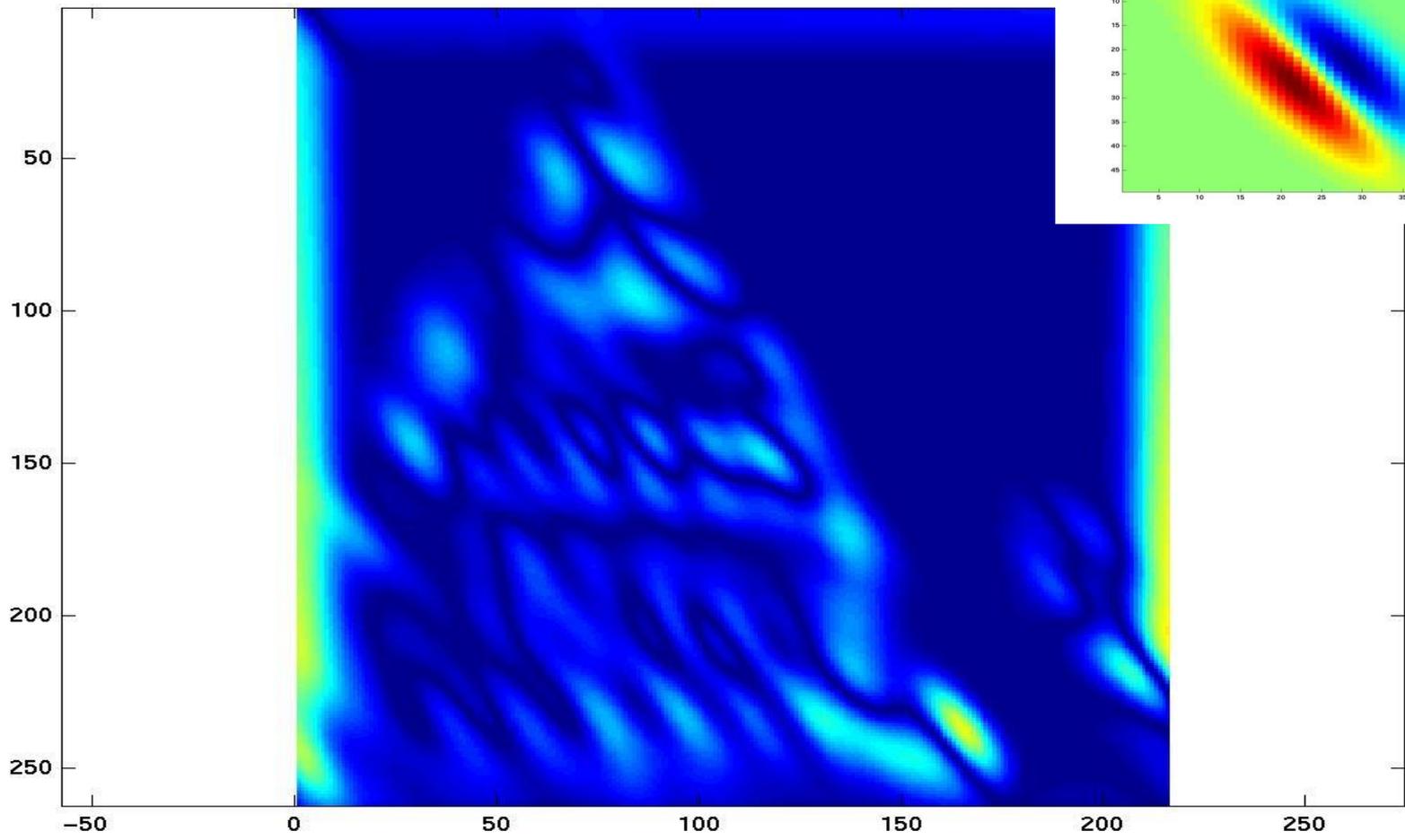


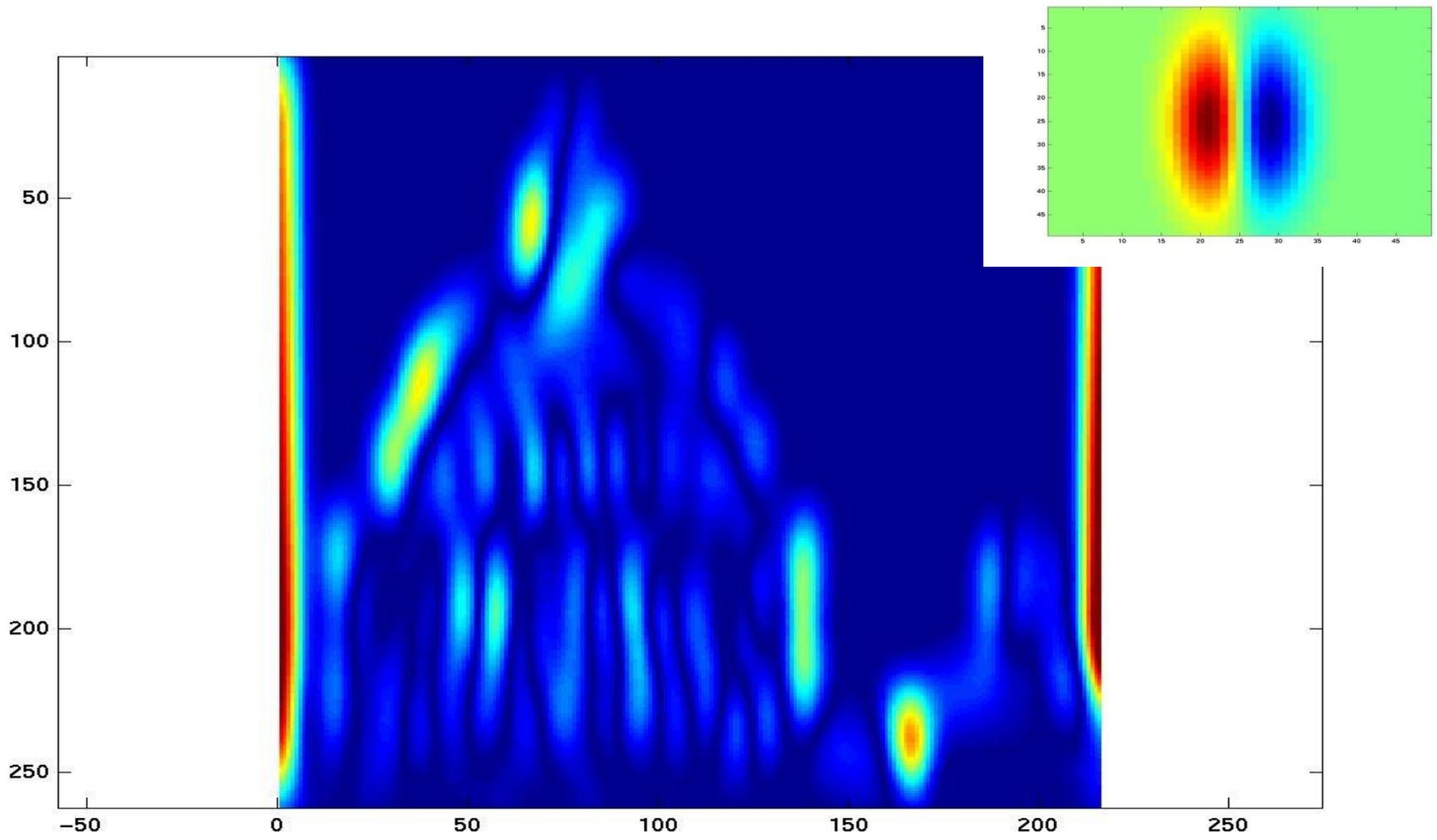


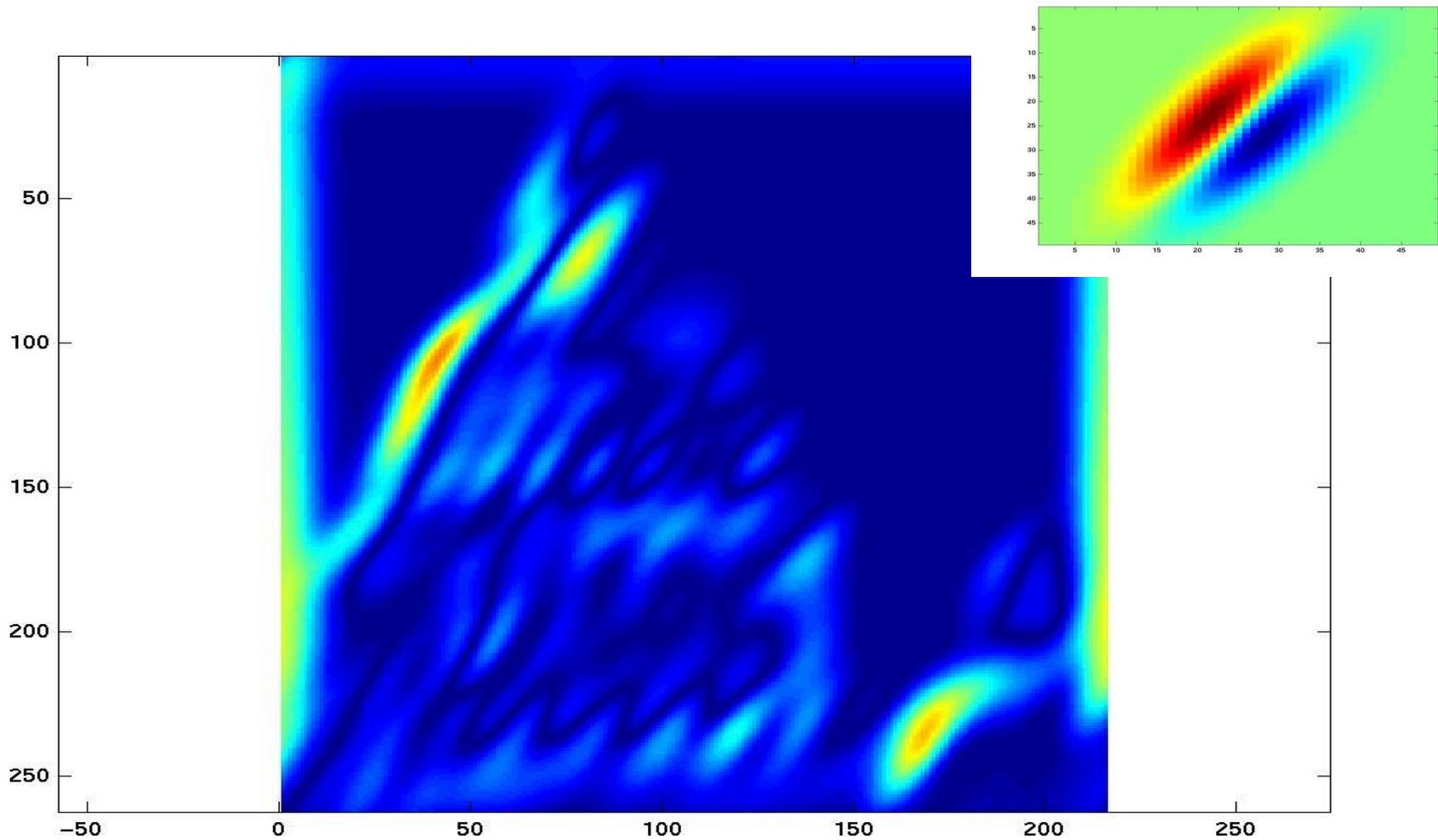


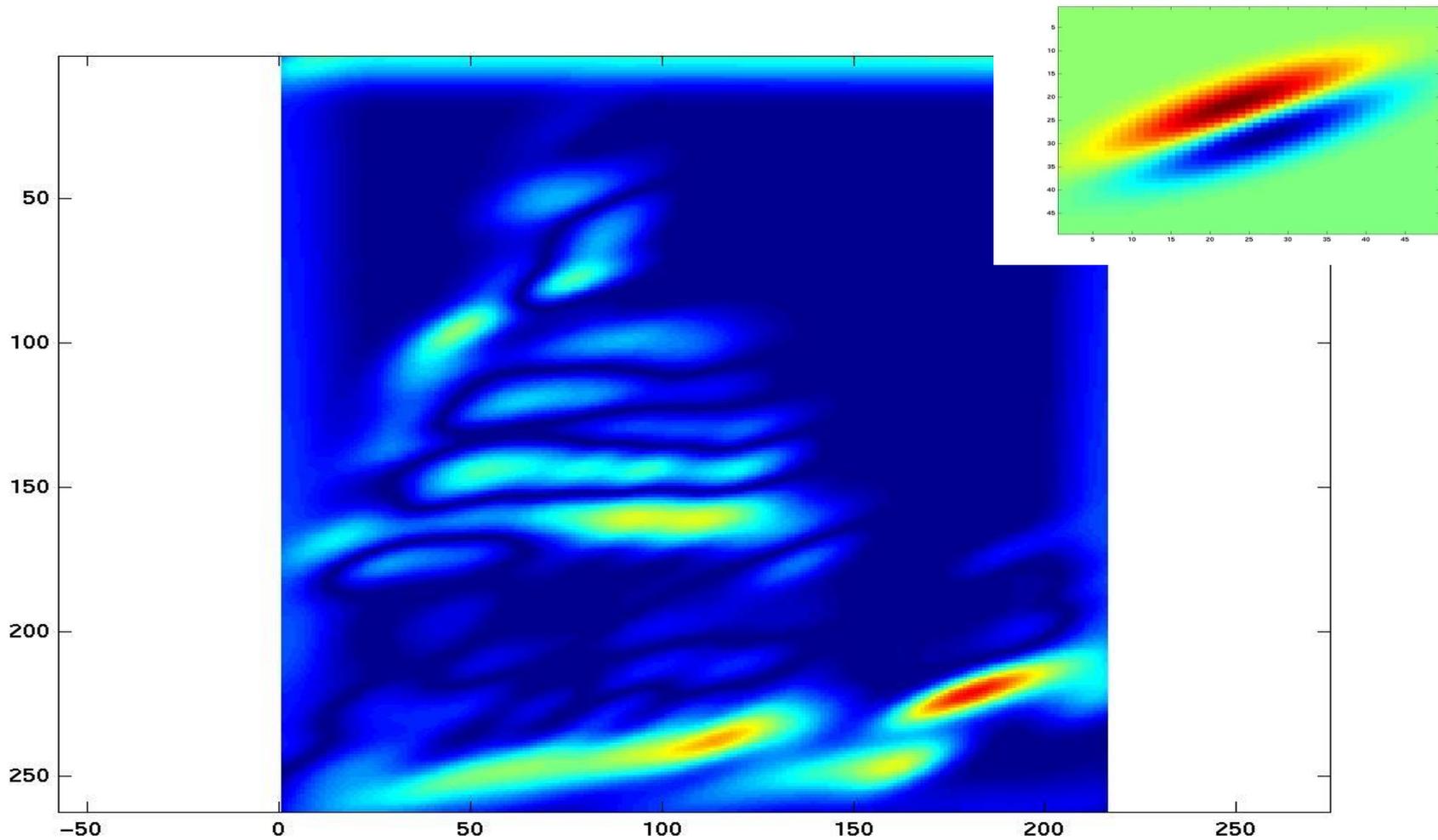






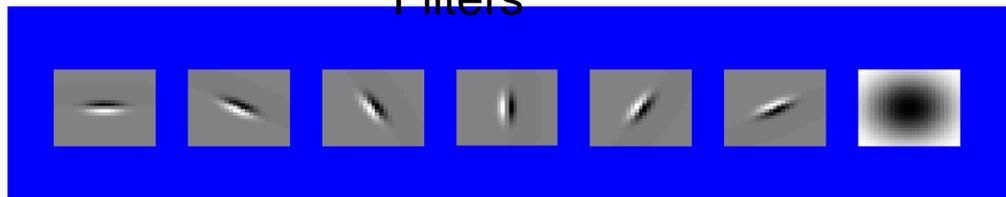




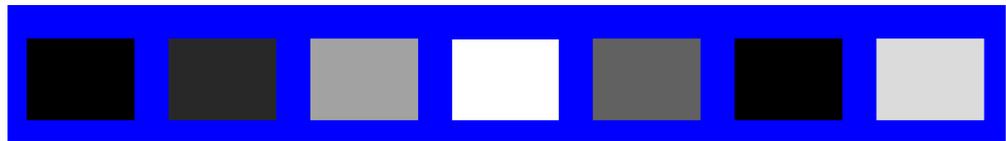


Can you match the texture to the response?

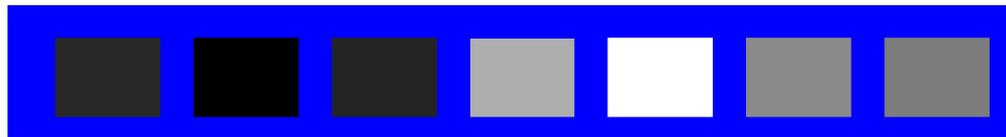
Filters



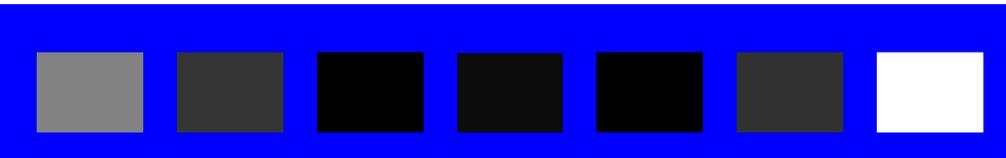
1



2

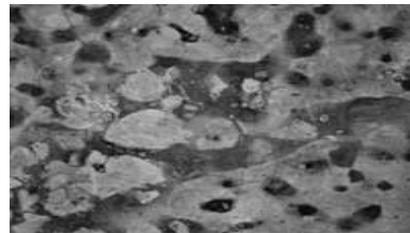


3



Mean abs responses

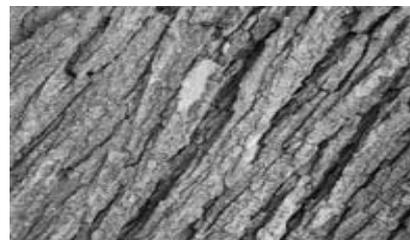
A



B



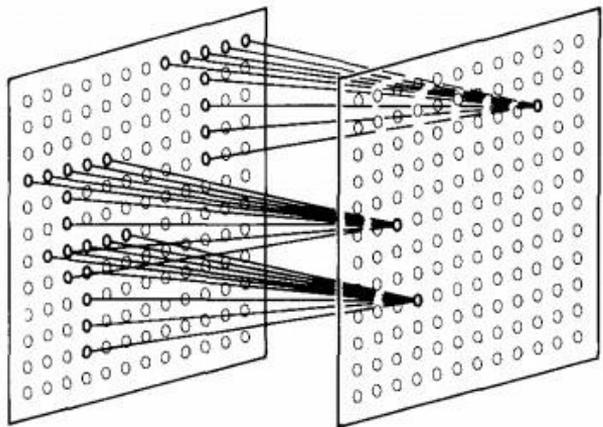
C



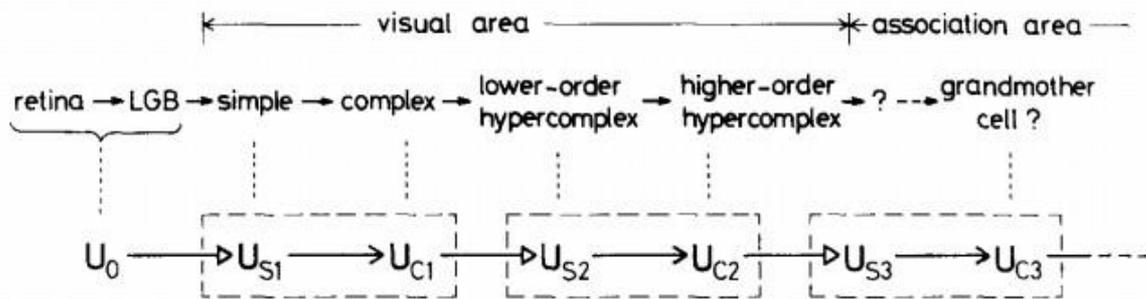
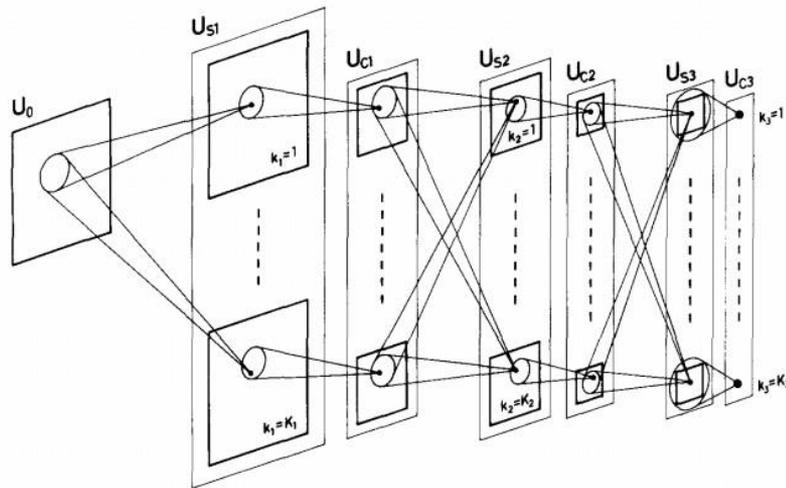
Meanwhile...

Neurocognitron

[Fukushima 1980]

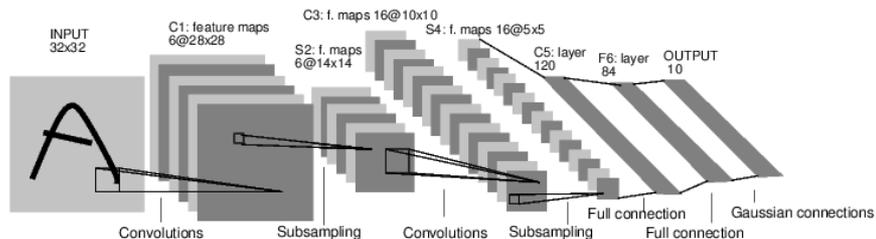


“sandwich” architecture (SCSCSC...)
simple cells: modifiable parameters
complex cells: perform pooling

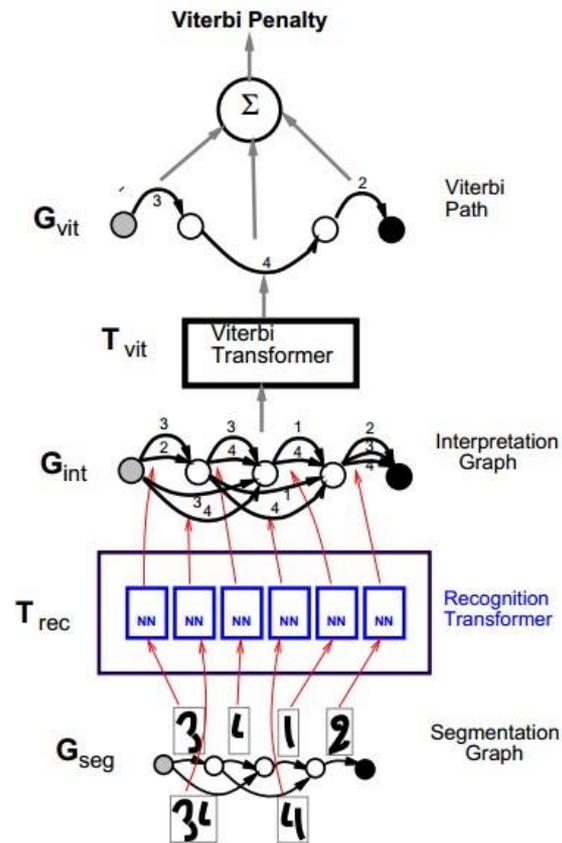


A bit of history: Gradient-based learning applied to document recognition

[LeCun, Bottou, Bengio, Haffner
1998]



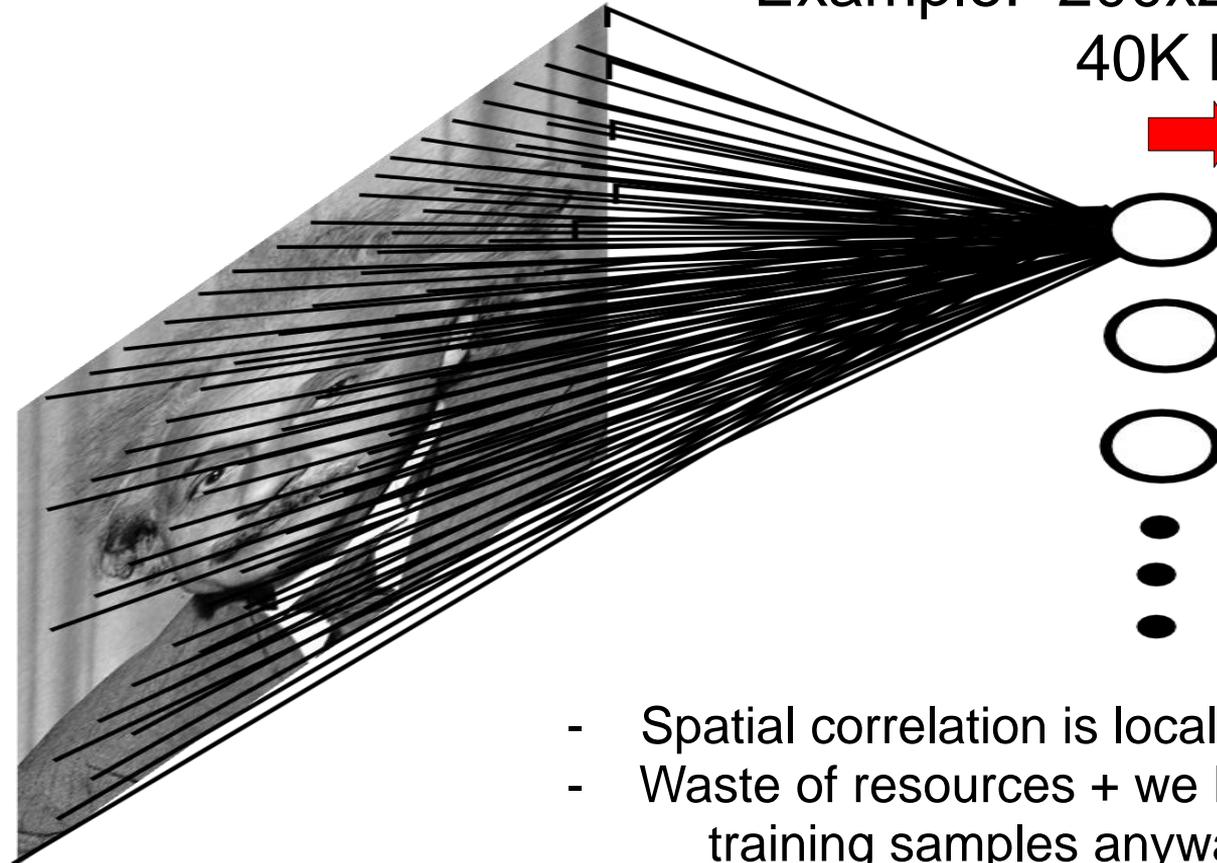
LeNet-5



Fully Connected Layer

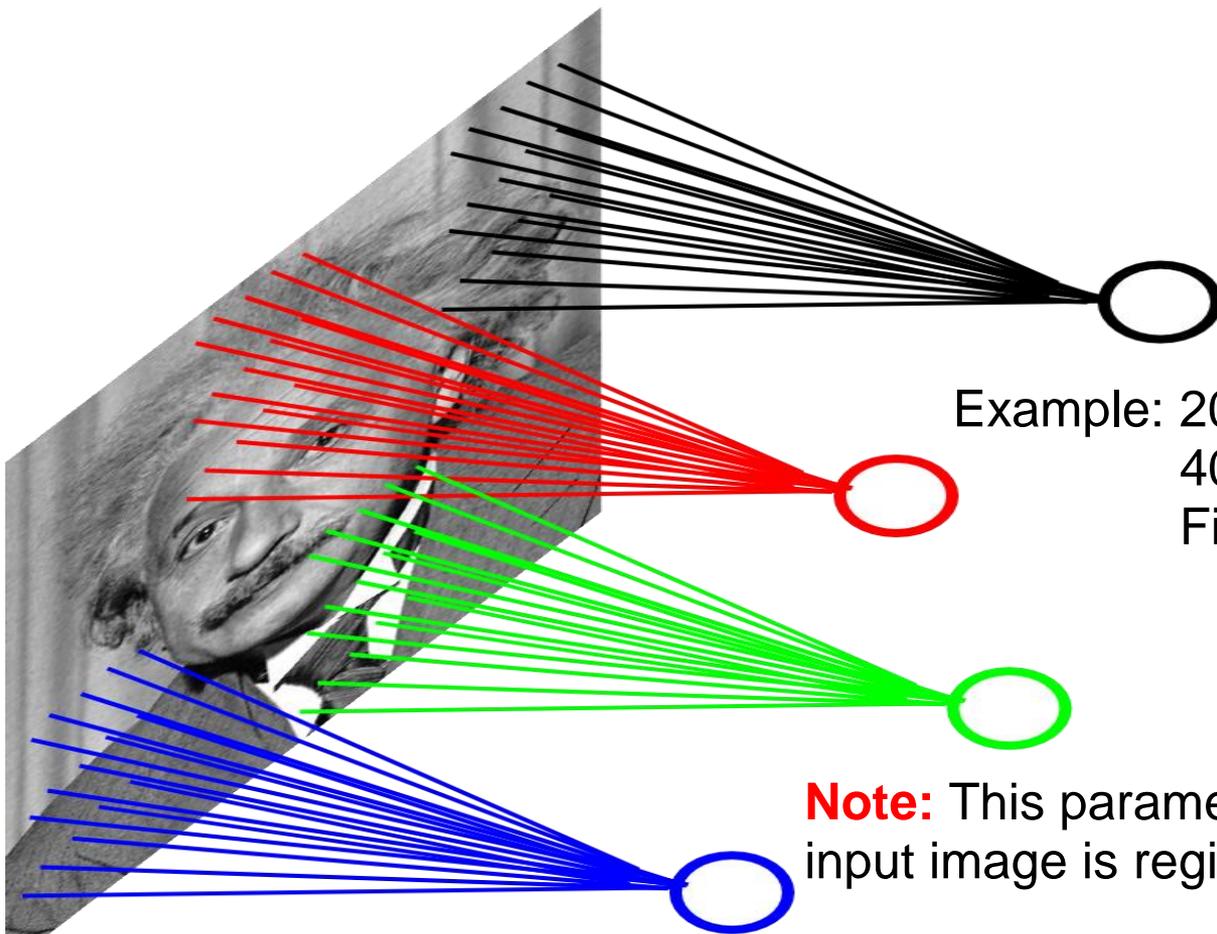
Example: 200x200 image
40K hidden units

➔ **~2B parameters!!!**



- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Locally Connected Layer

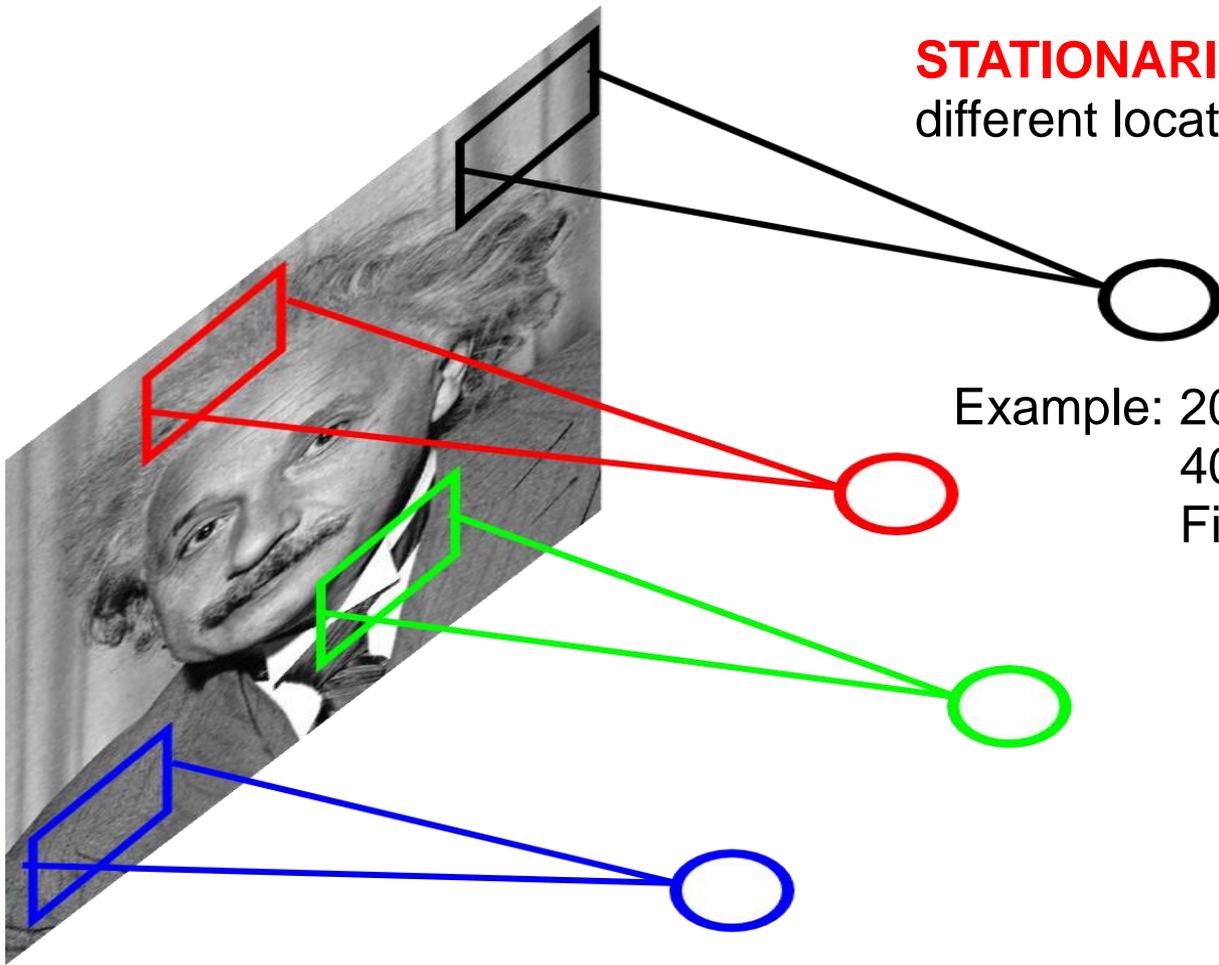


Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

Note: This parameterization is good when input image is registered (e.g., face recognition).

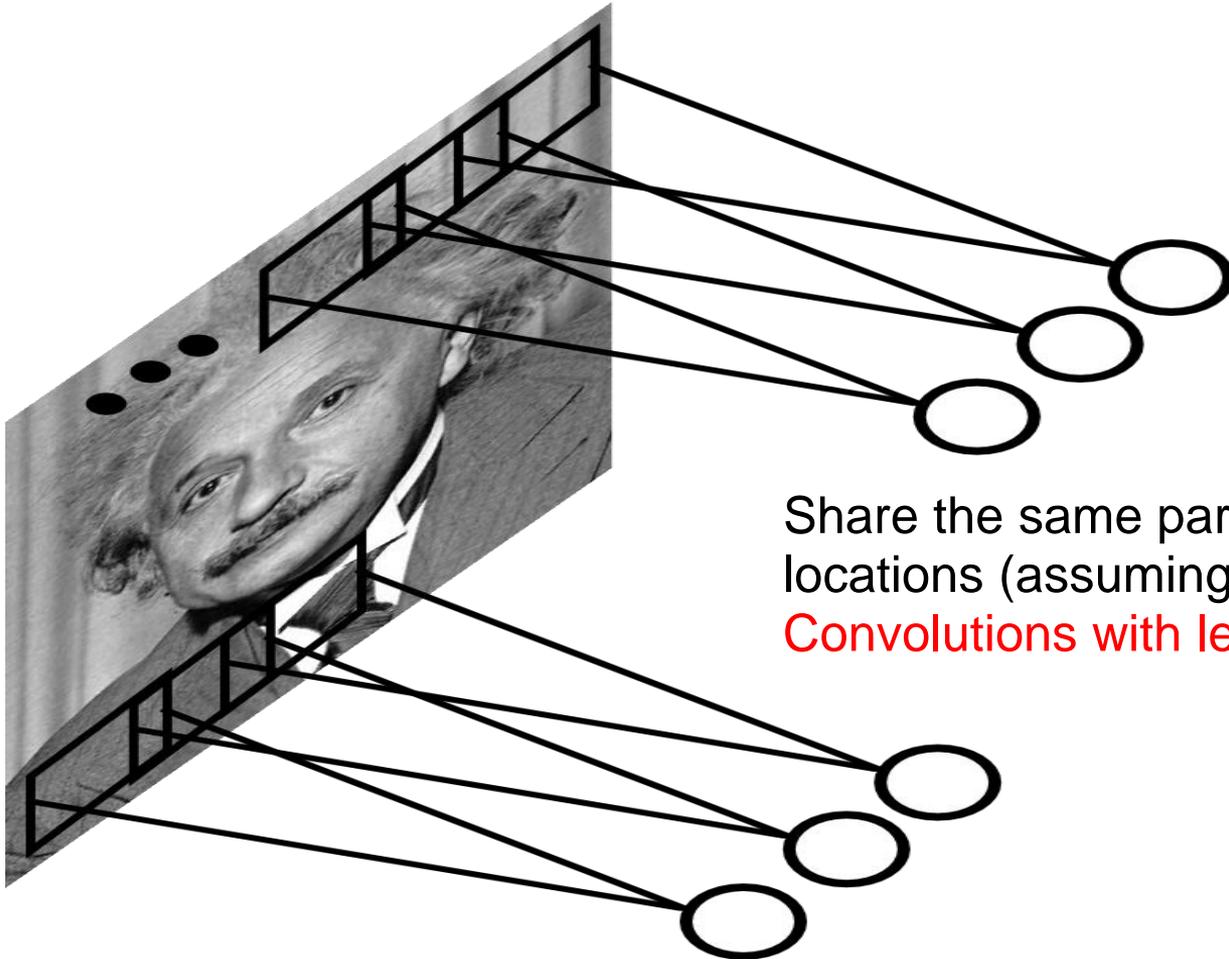
Locally Connected Layer

STATIONARITY? Statistics is similar at different locations



Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

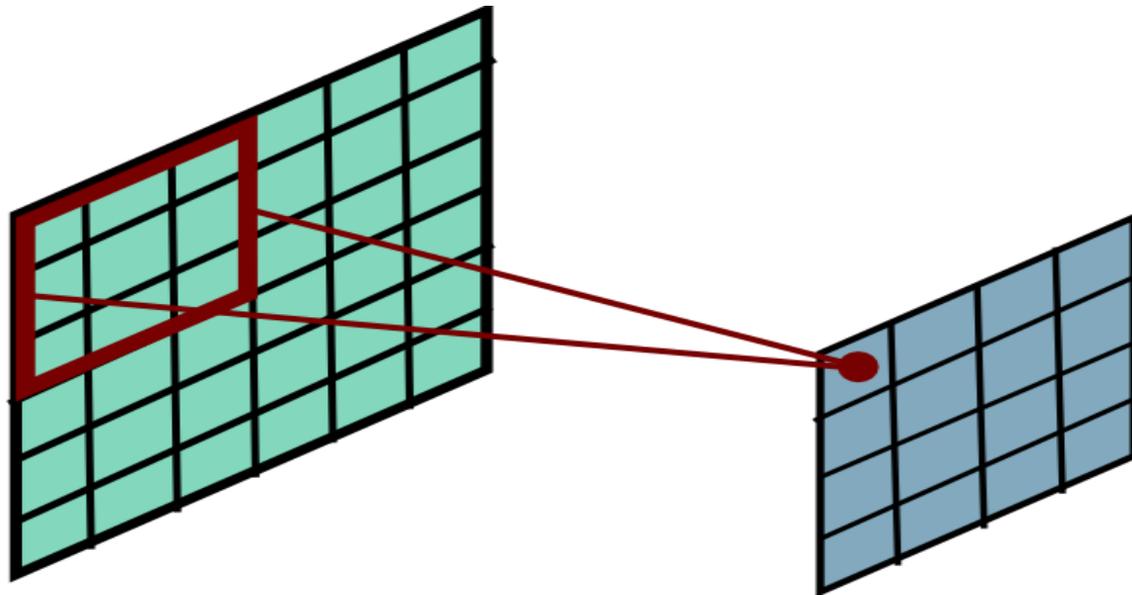
Convolutional Layer



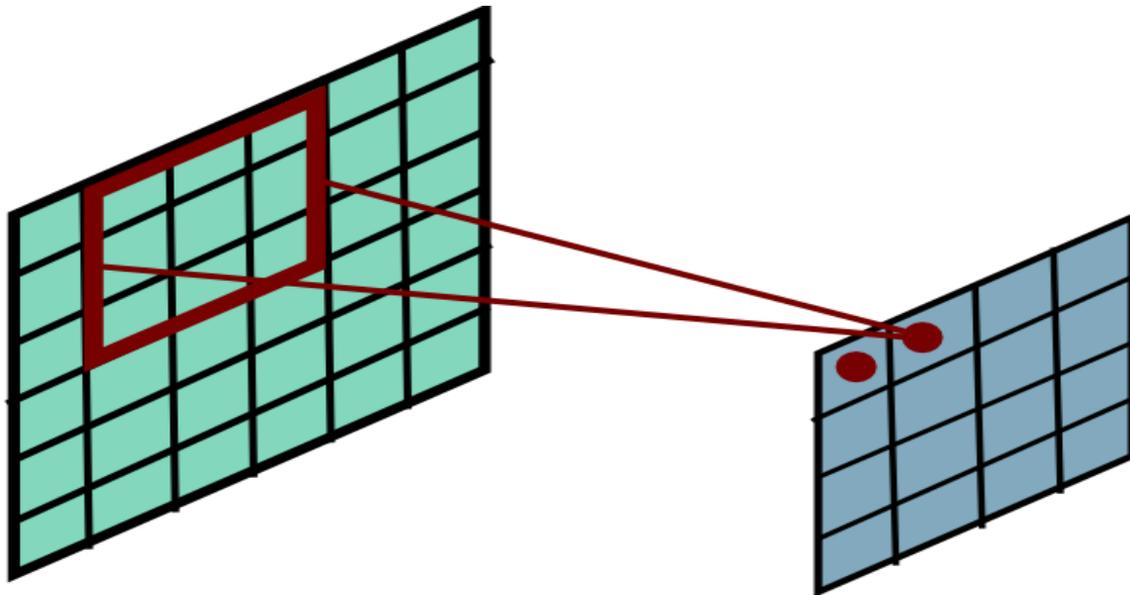
Share the same parameters across different locations (assuming input is stationary):

Convolutions with learned kernels

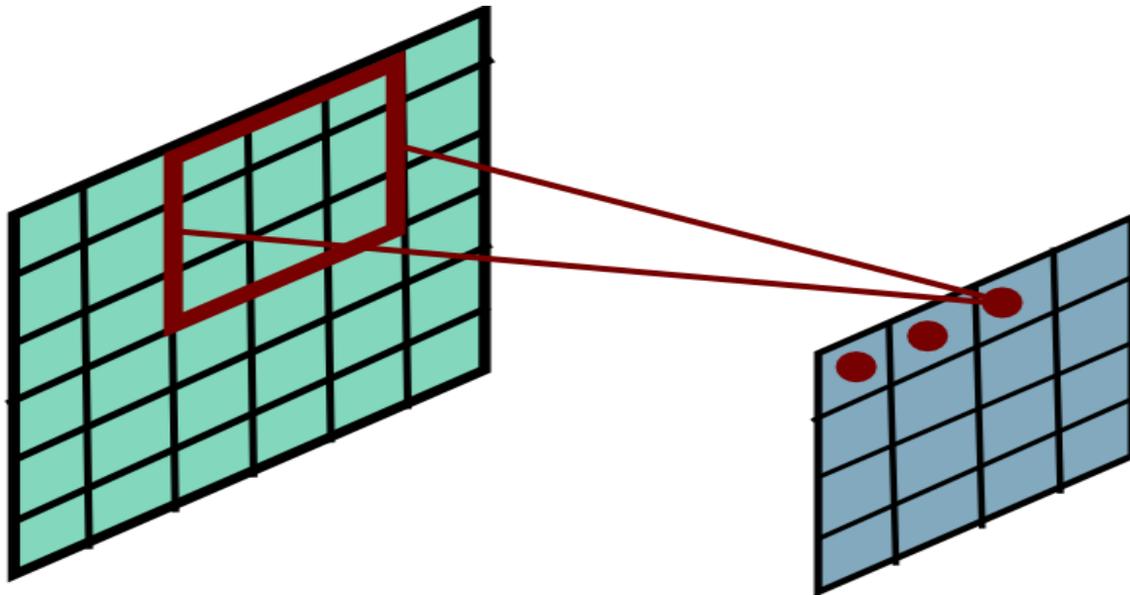
Convolutional Layer



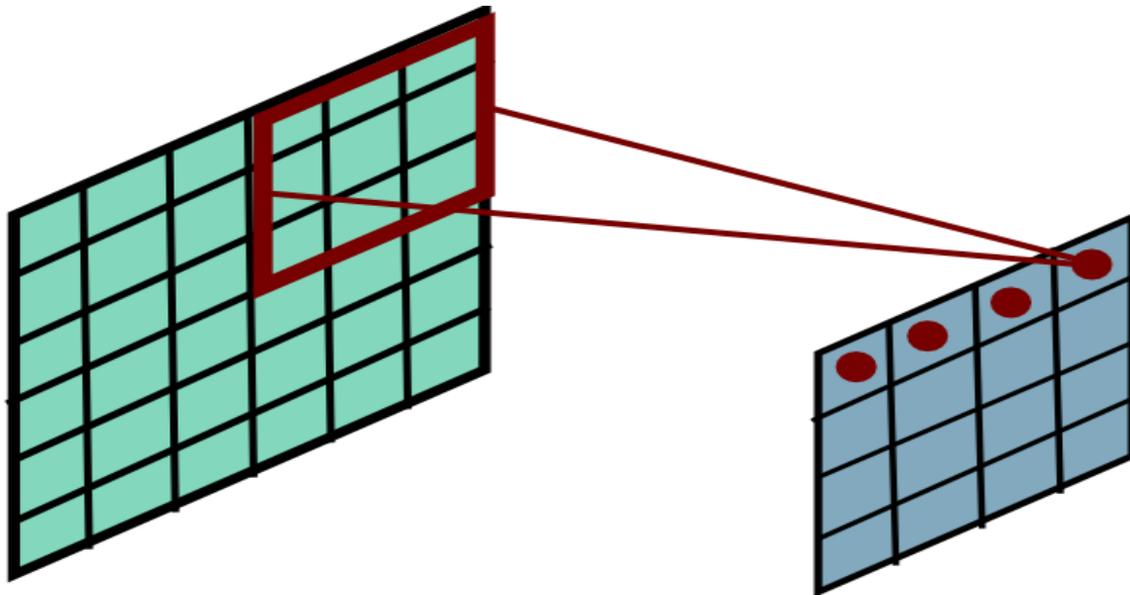
Convolutional Layer



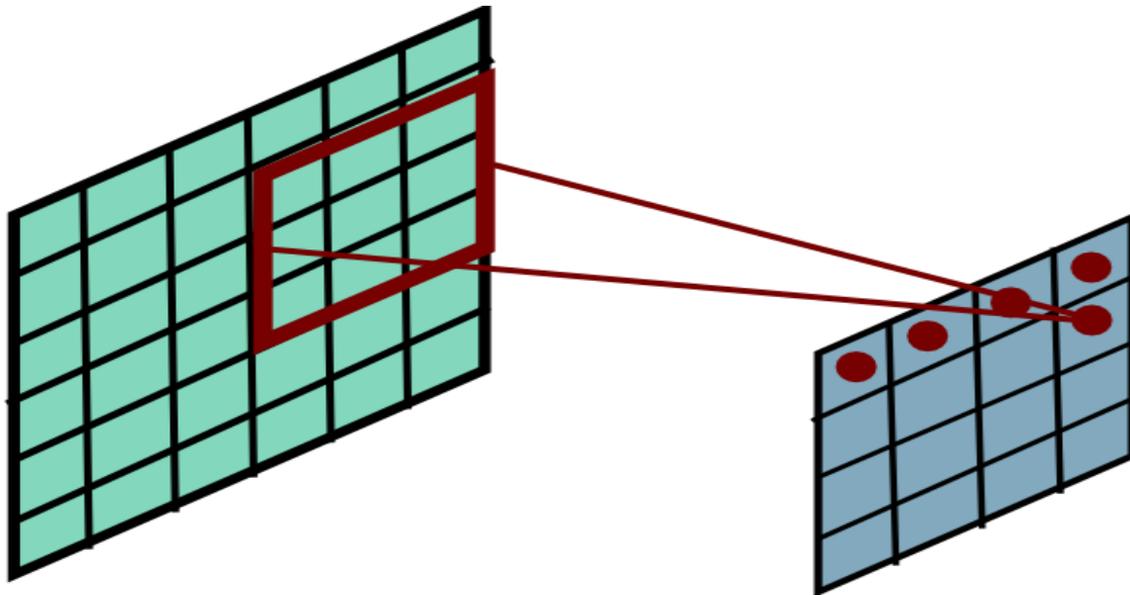
Convolutional Layer



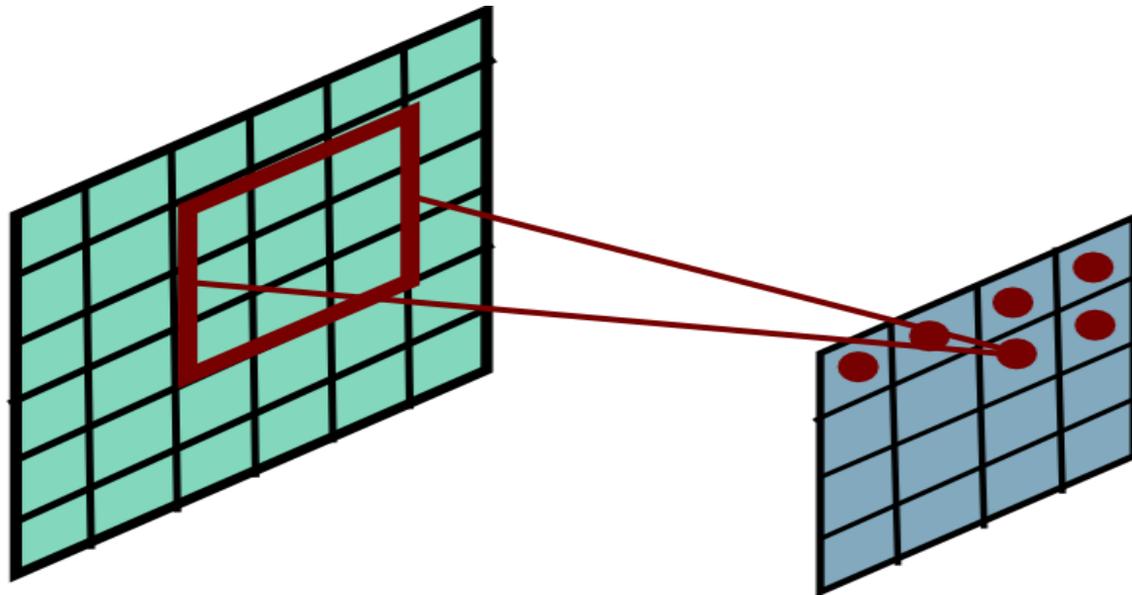
Convolutional Layer



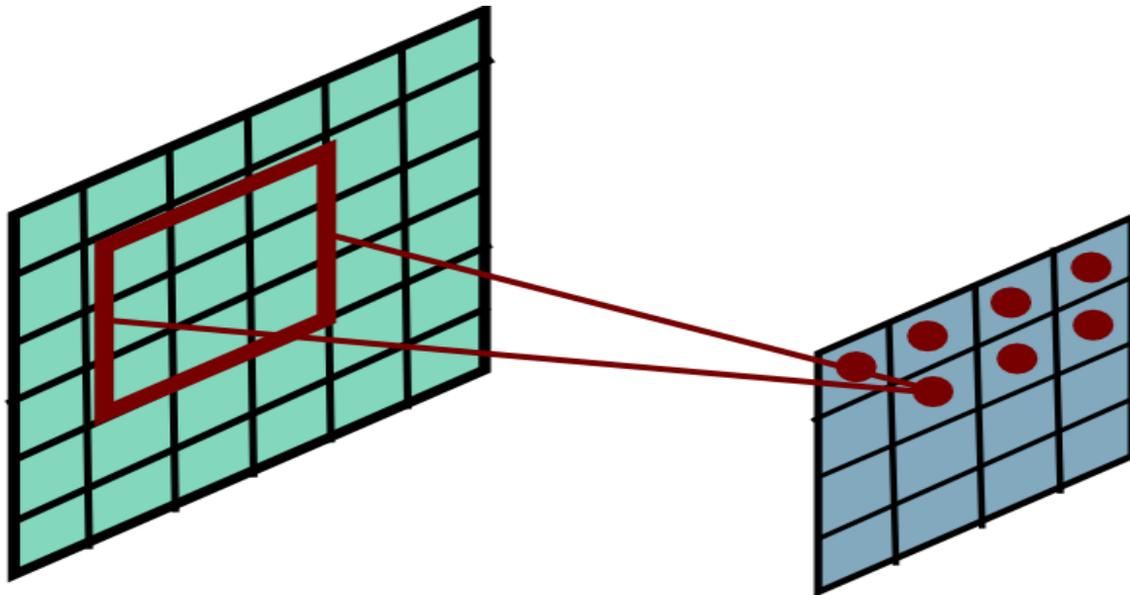
Convolutional Layer



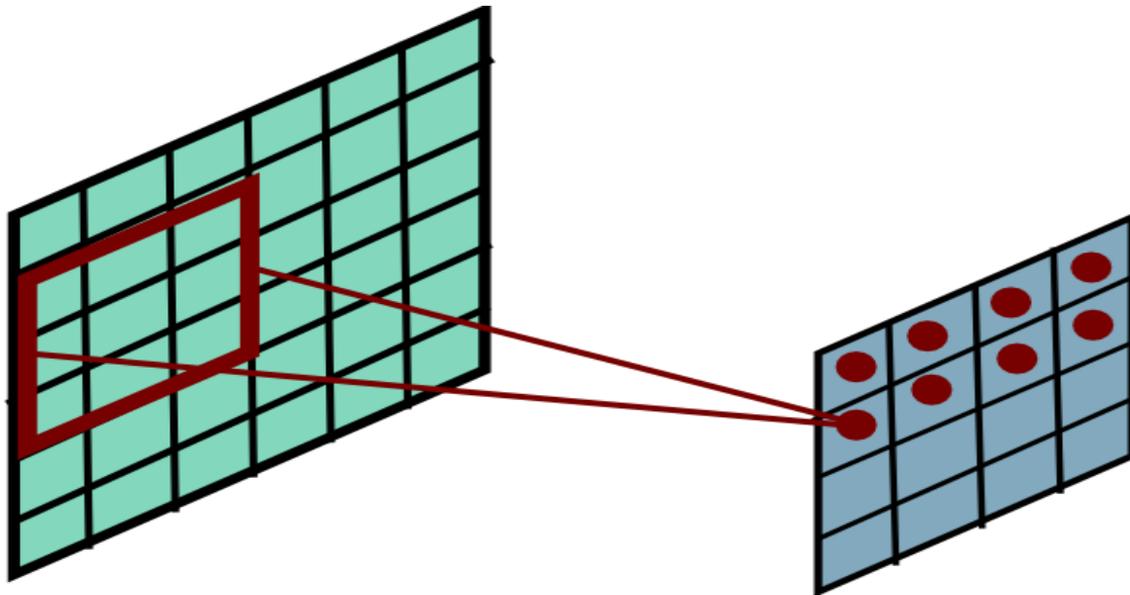
Convolutional Layer



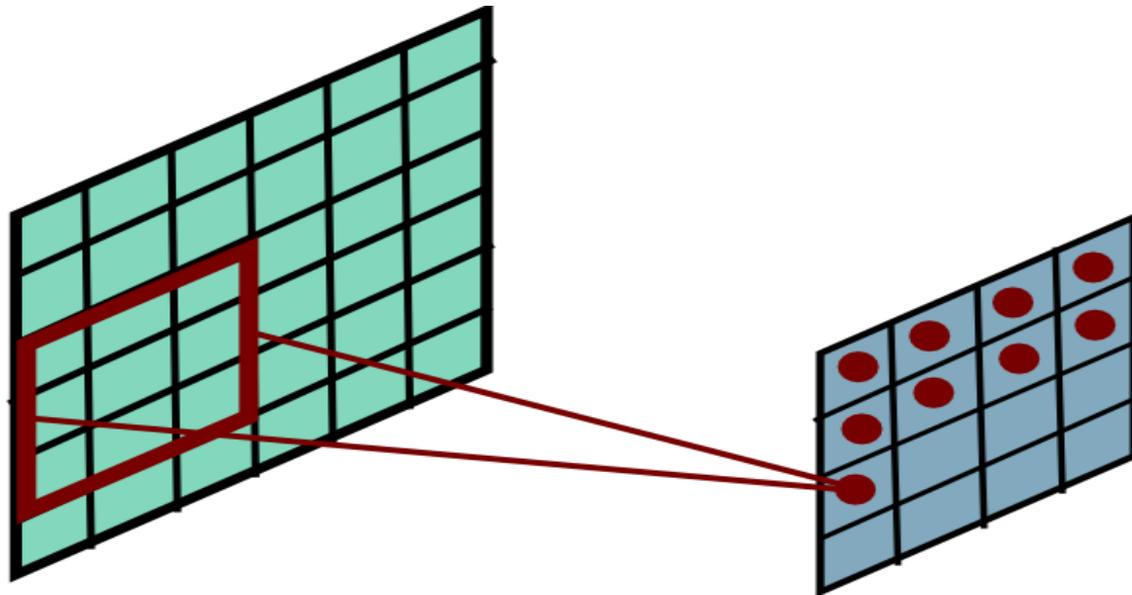
Convolutional Layer



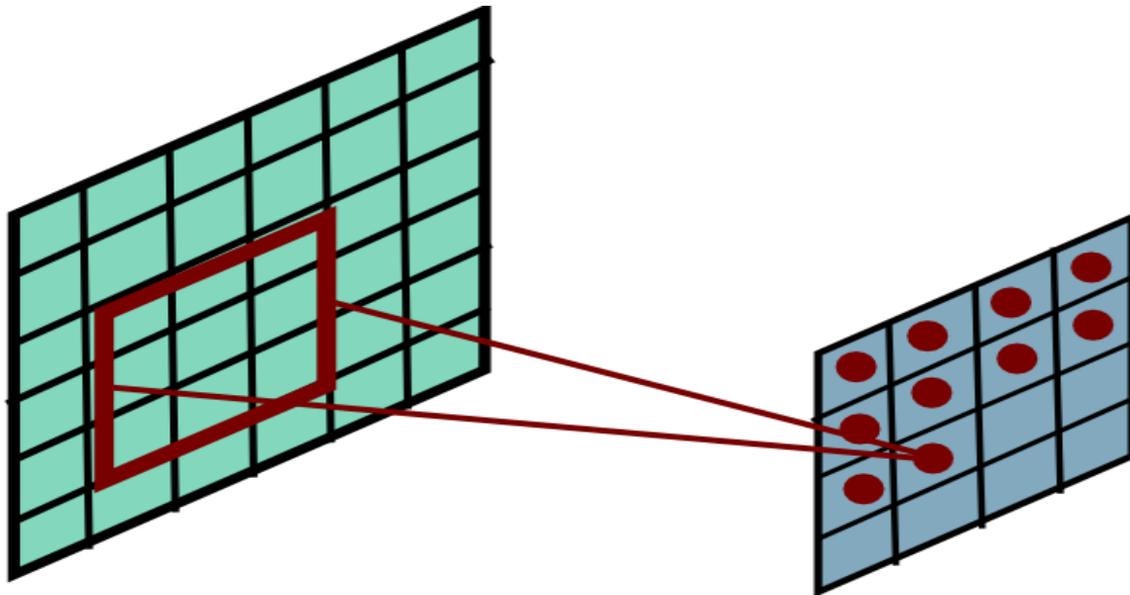
Convolutional Layer



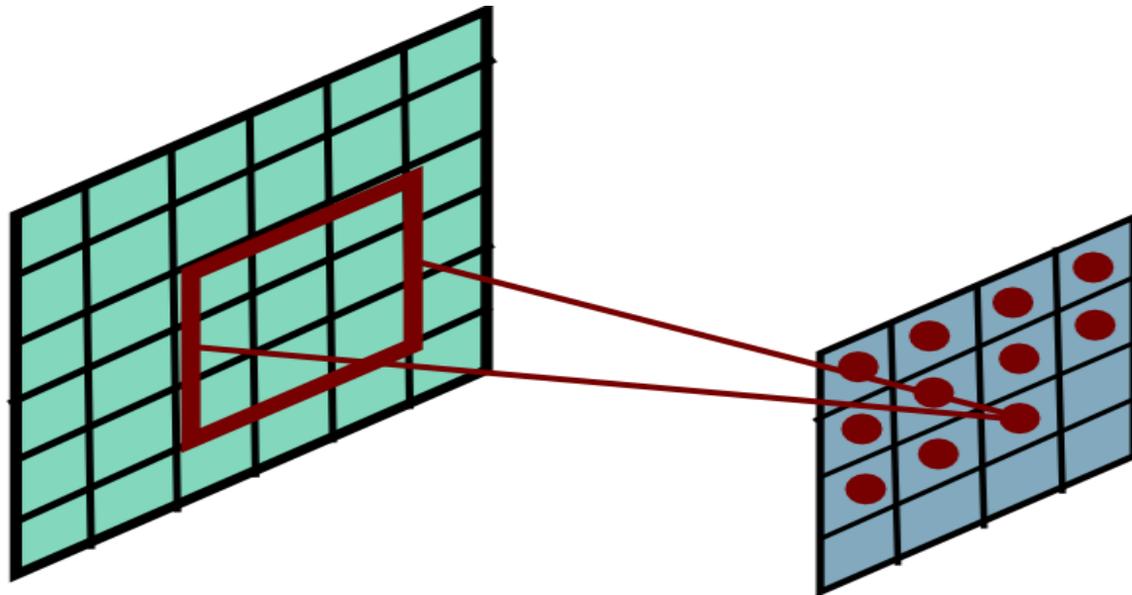
Convolutional Layer



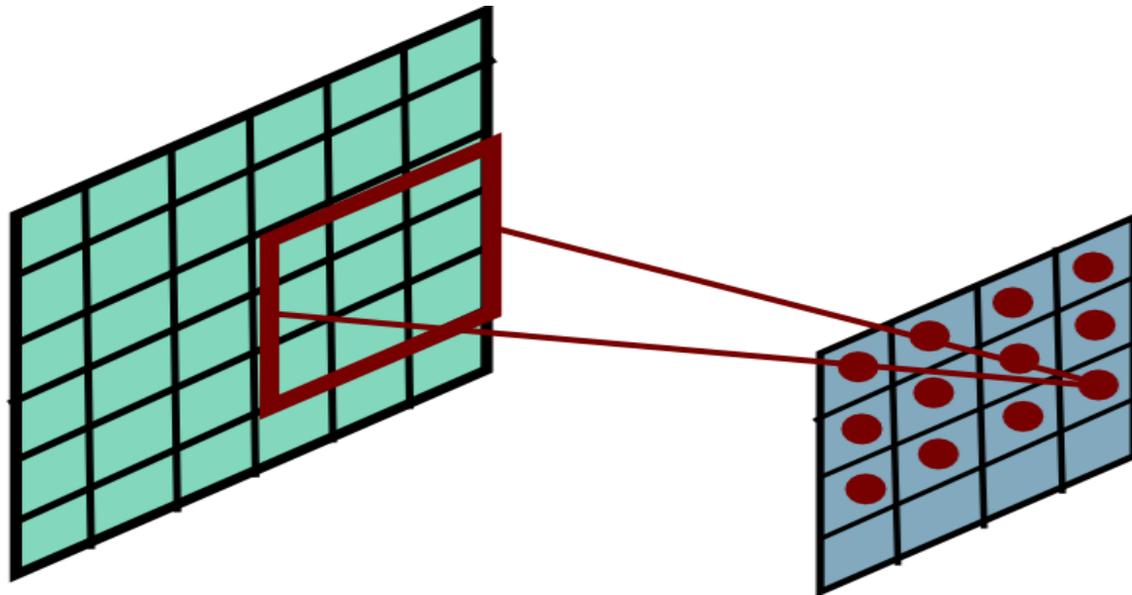
Convolutional Layer



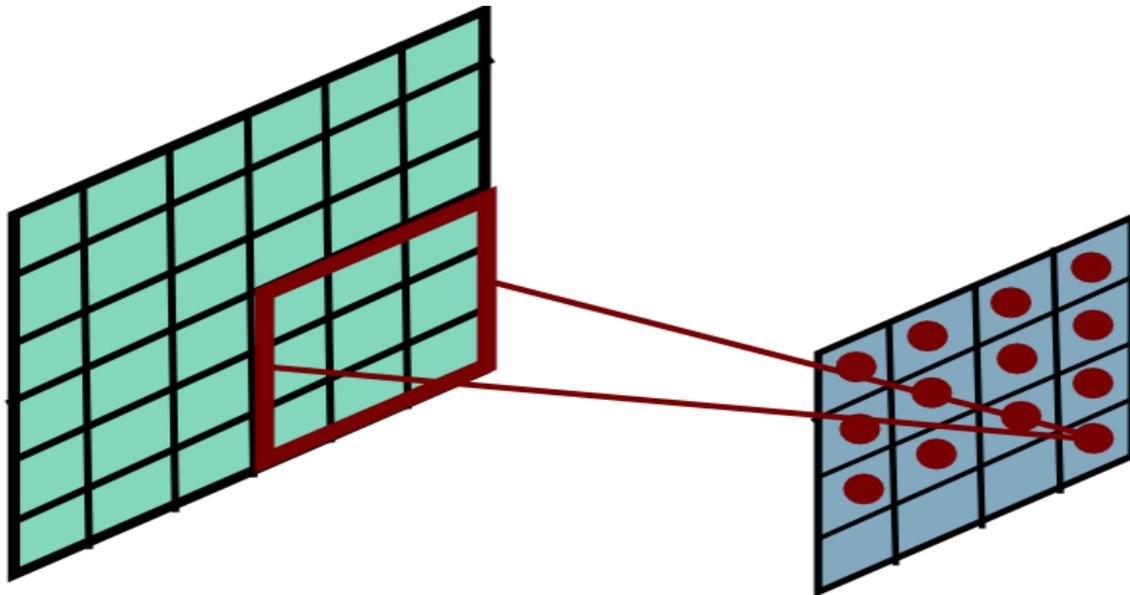
Convolutional Layer



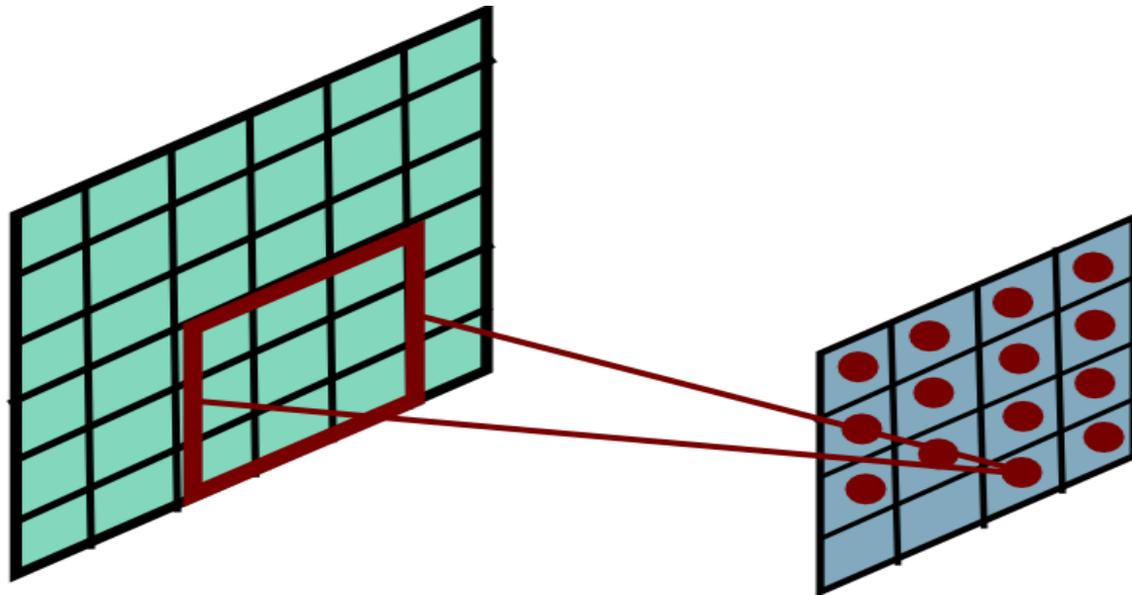
Convolutional Layer



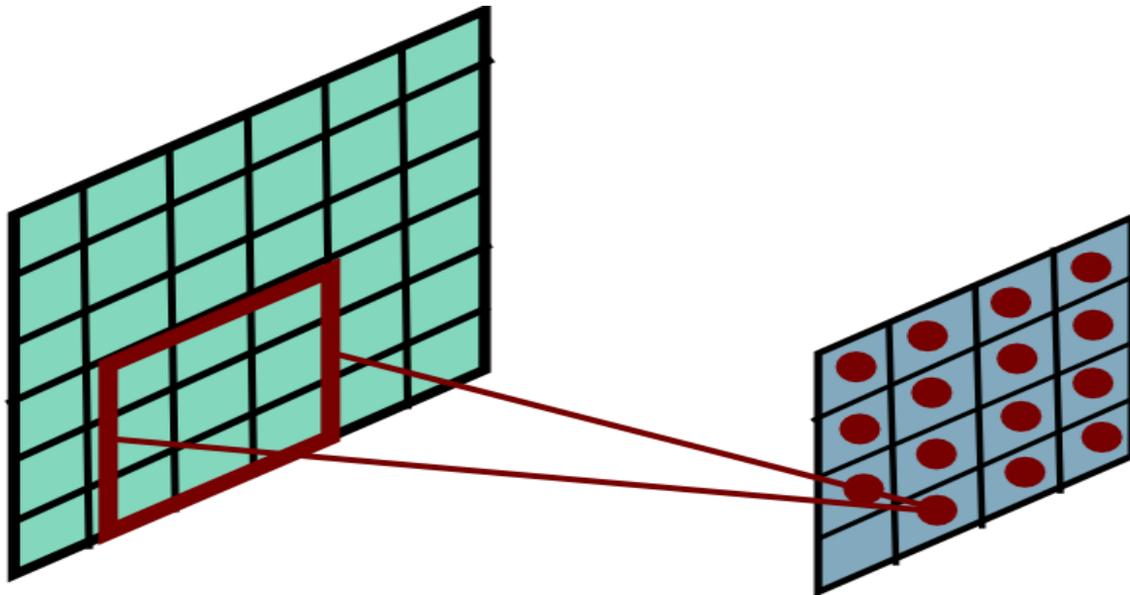
Convolutional Layer



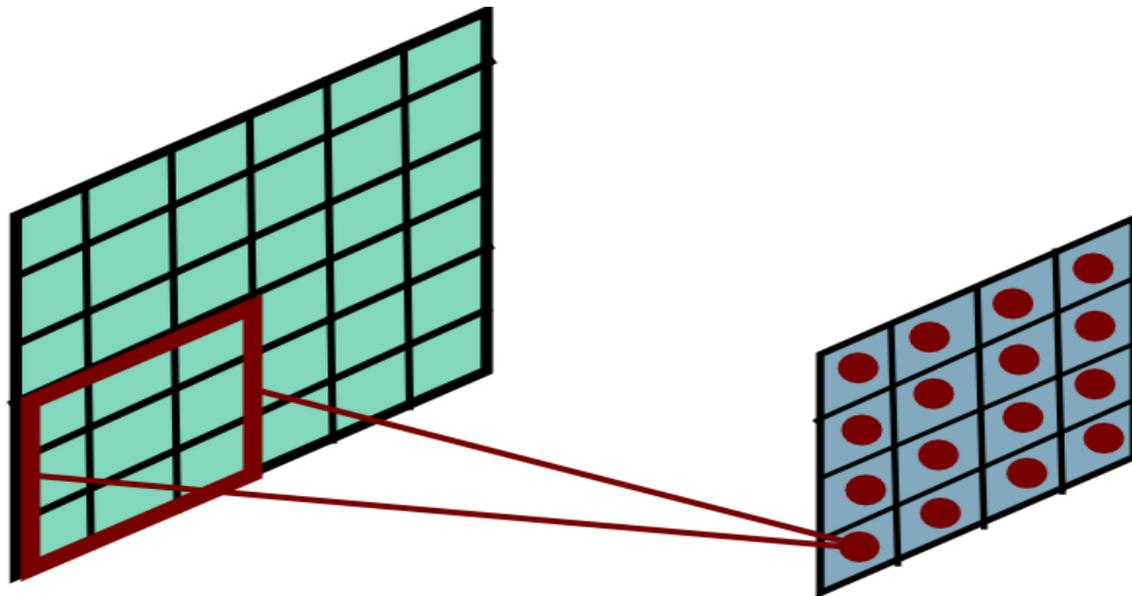
Convolutional Layer



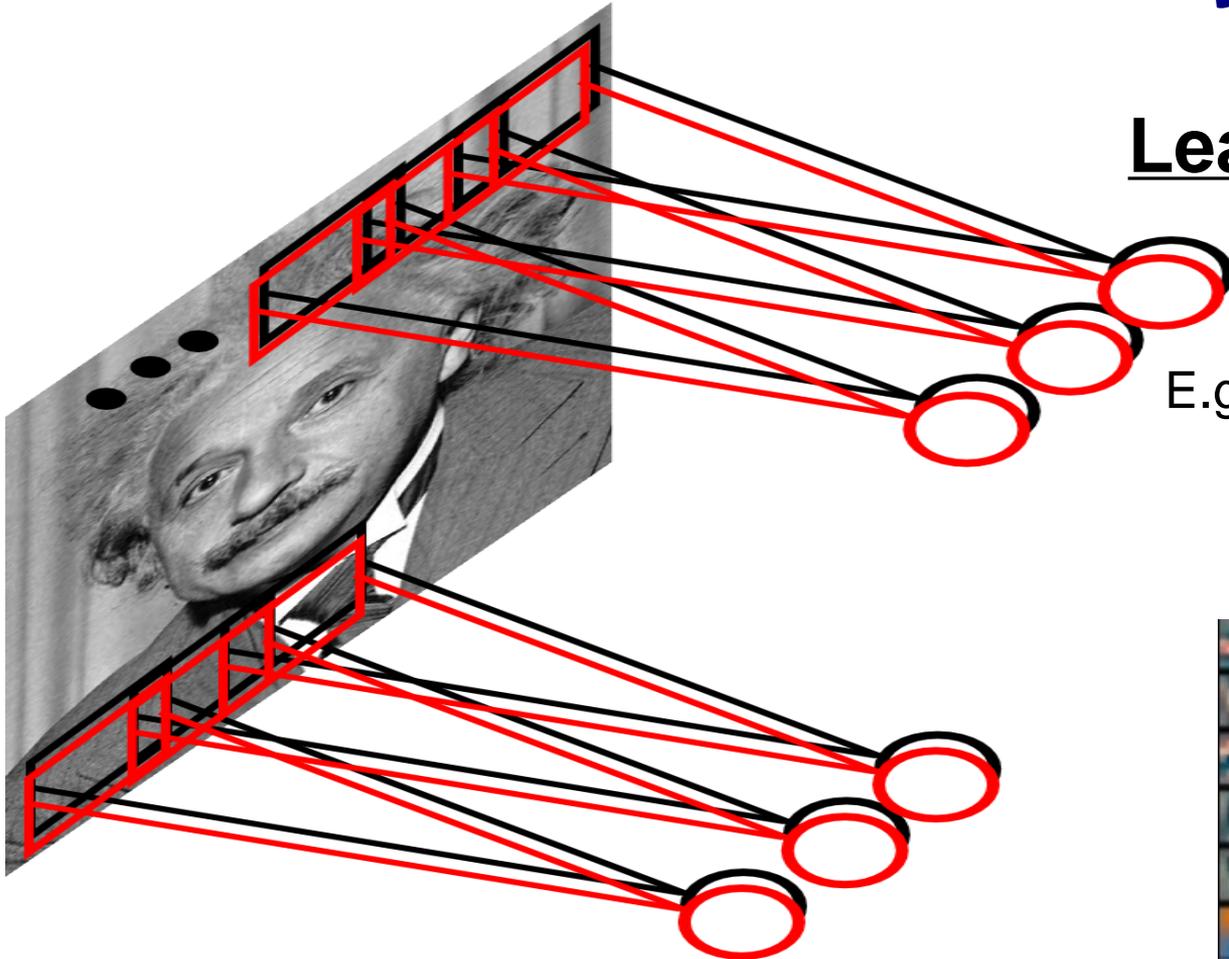
Convolutional Layer



Convolutional Layer

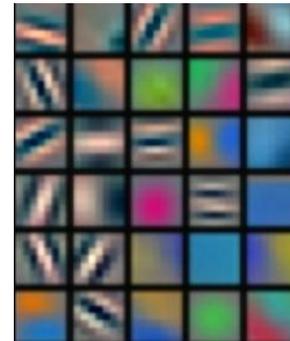


Convolutional Layer

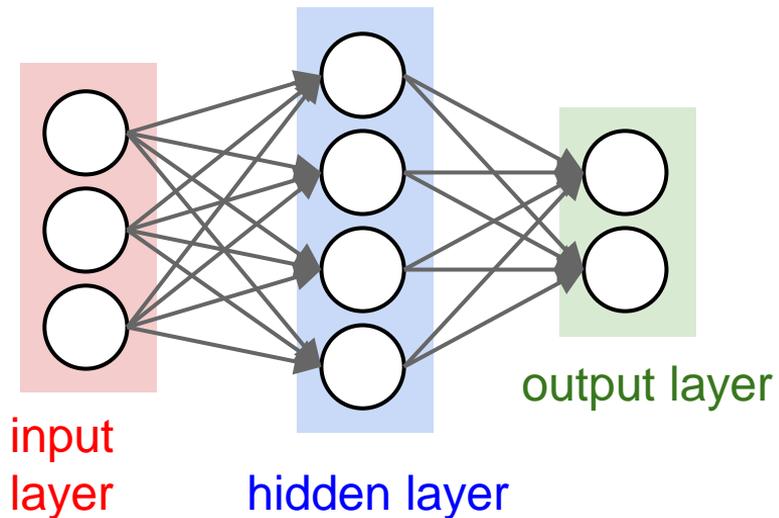


Learn multiple filters.

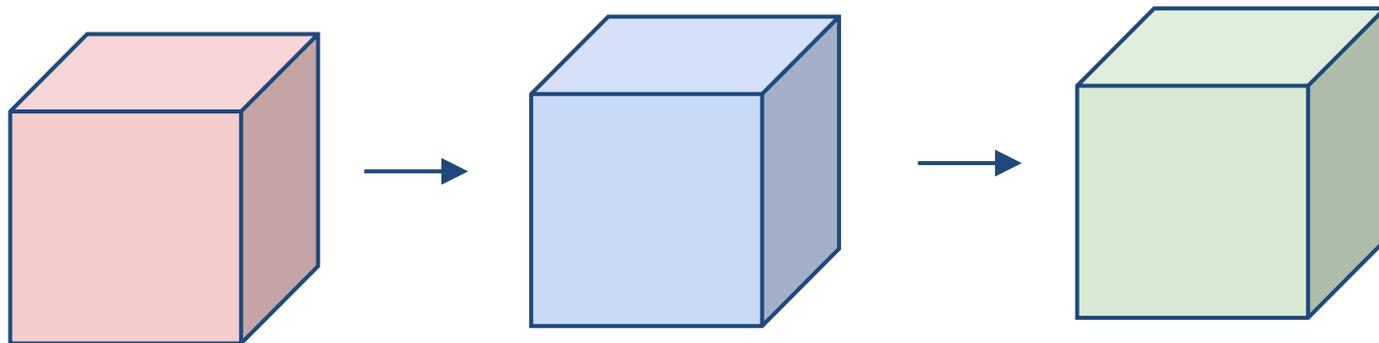
E.g.: 200x200 image
100 Filters
Filter size: 10x10
10K parameters



before:

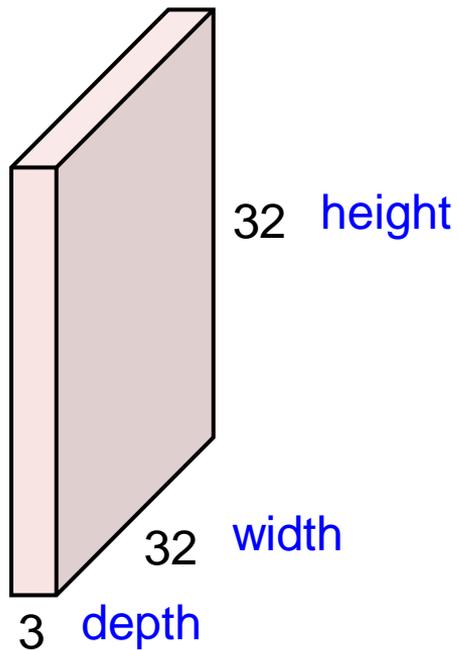


now:



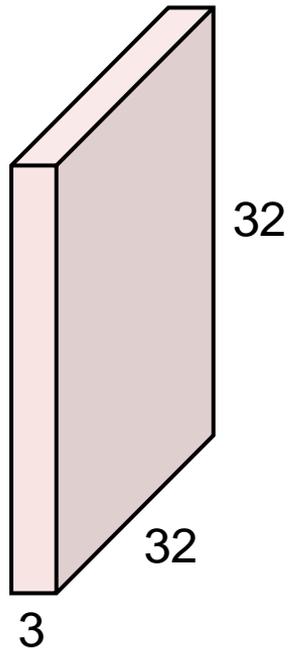
Convolution Layer

32x32x3 image

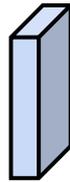


Convolution Layer

32x32x3 image



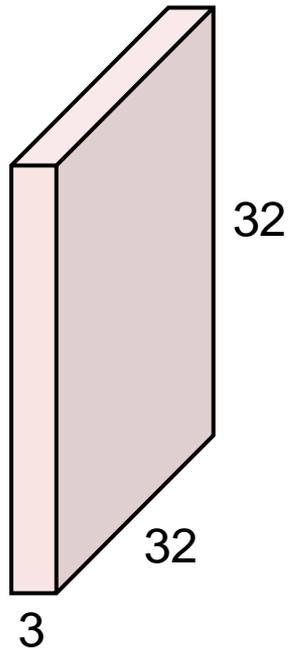
5x5x3 filter



Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

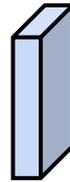
Convolution Layer

32x32x3 image



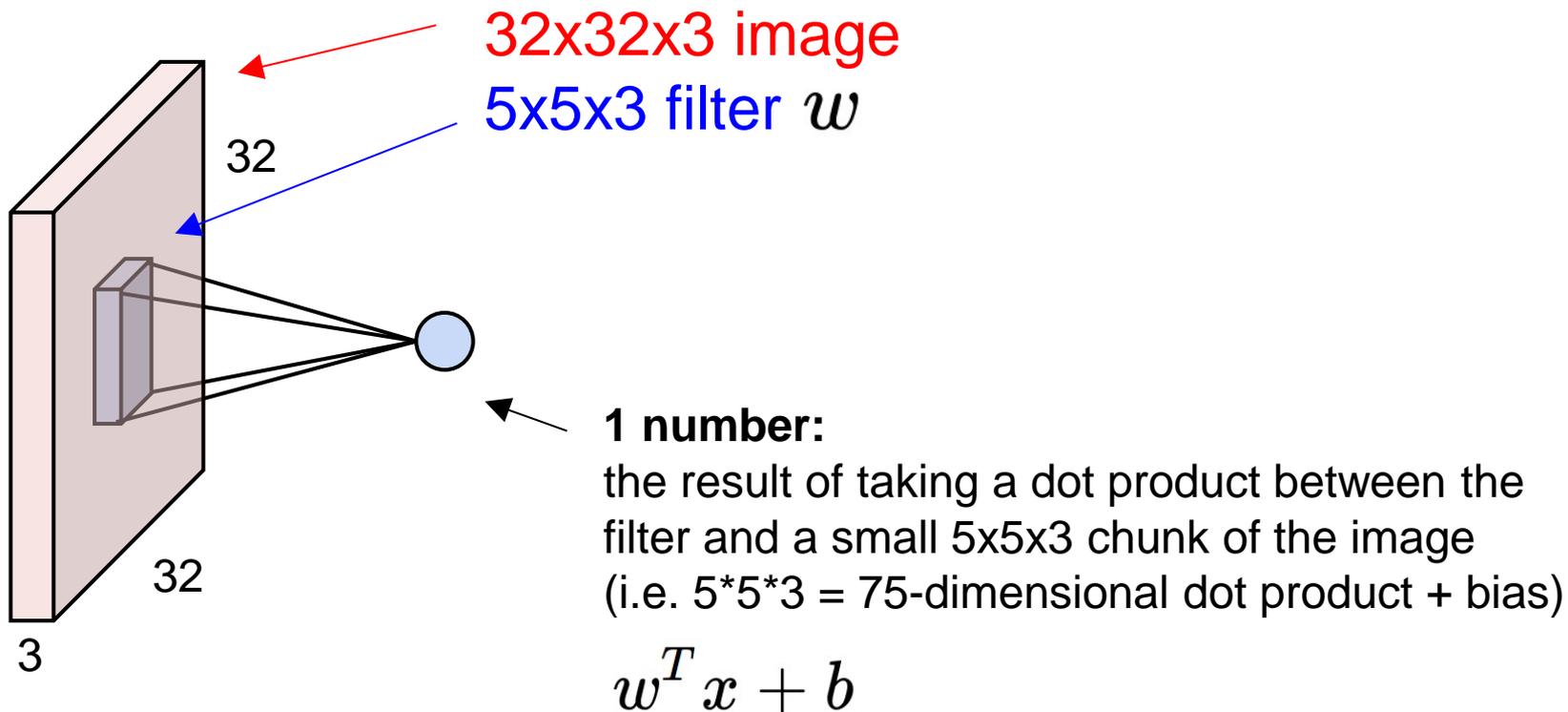
Filters always extend the full depth of the input volume

5x5x3 filter

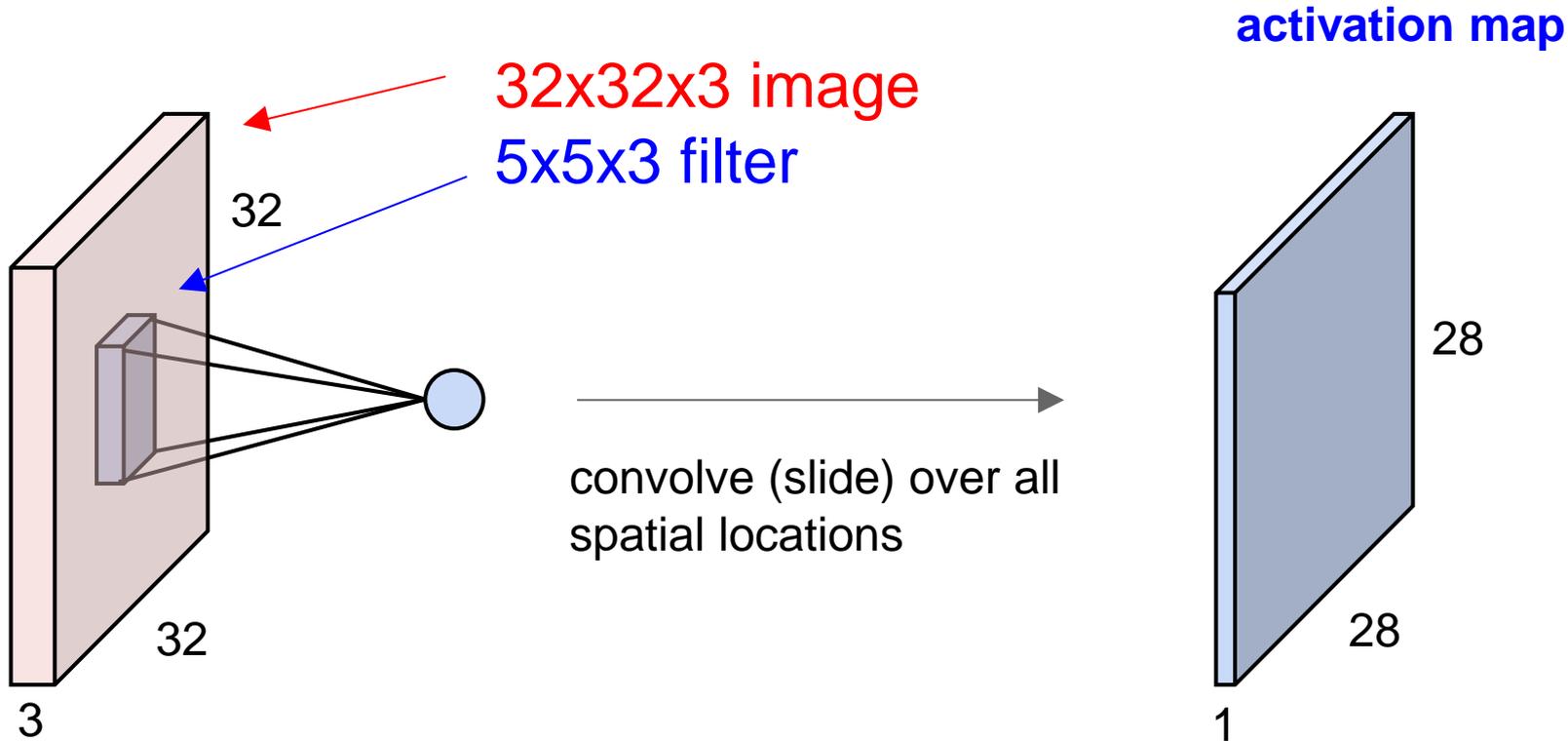


Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolution Layer

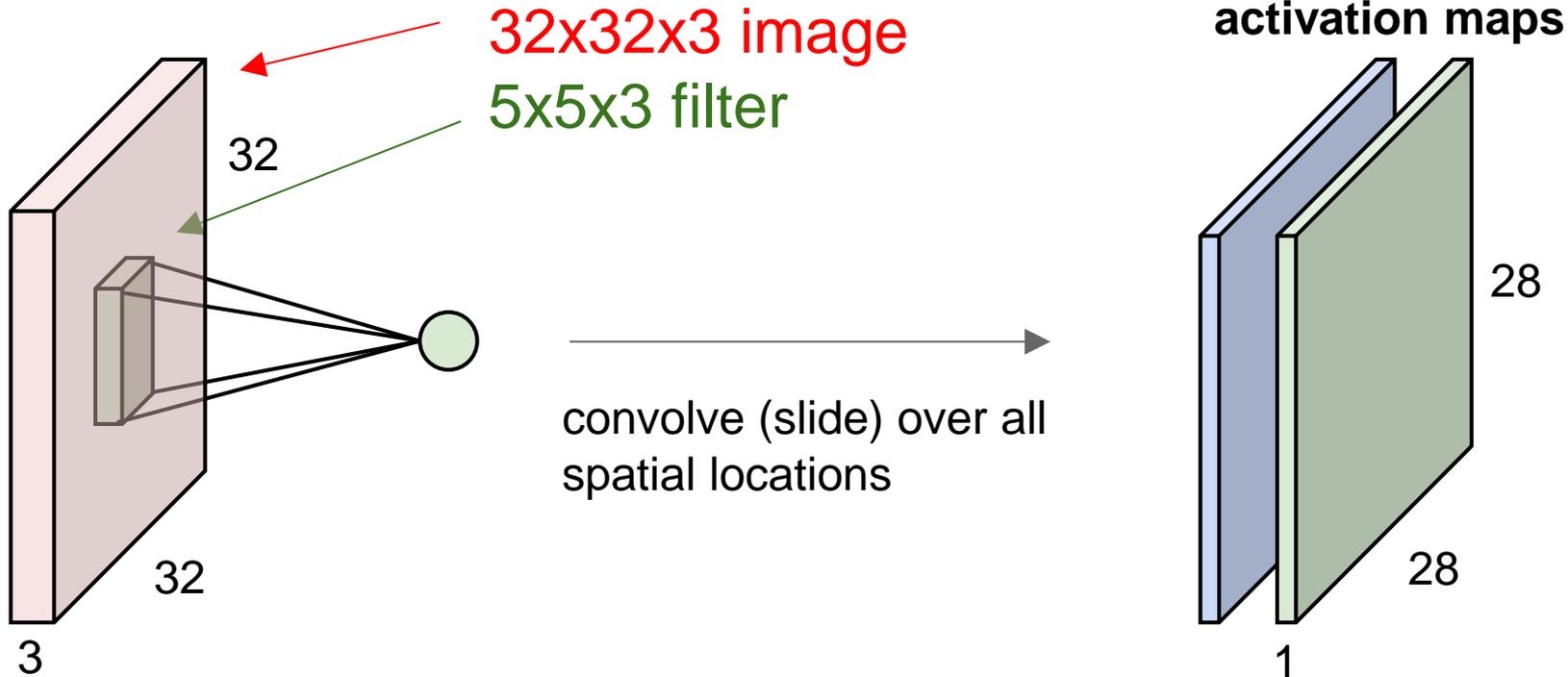


Convolution Layer

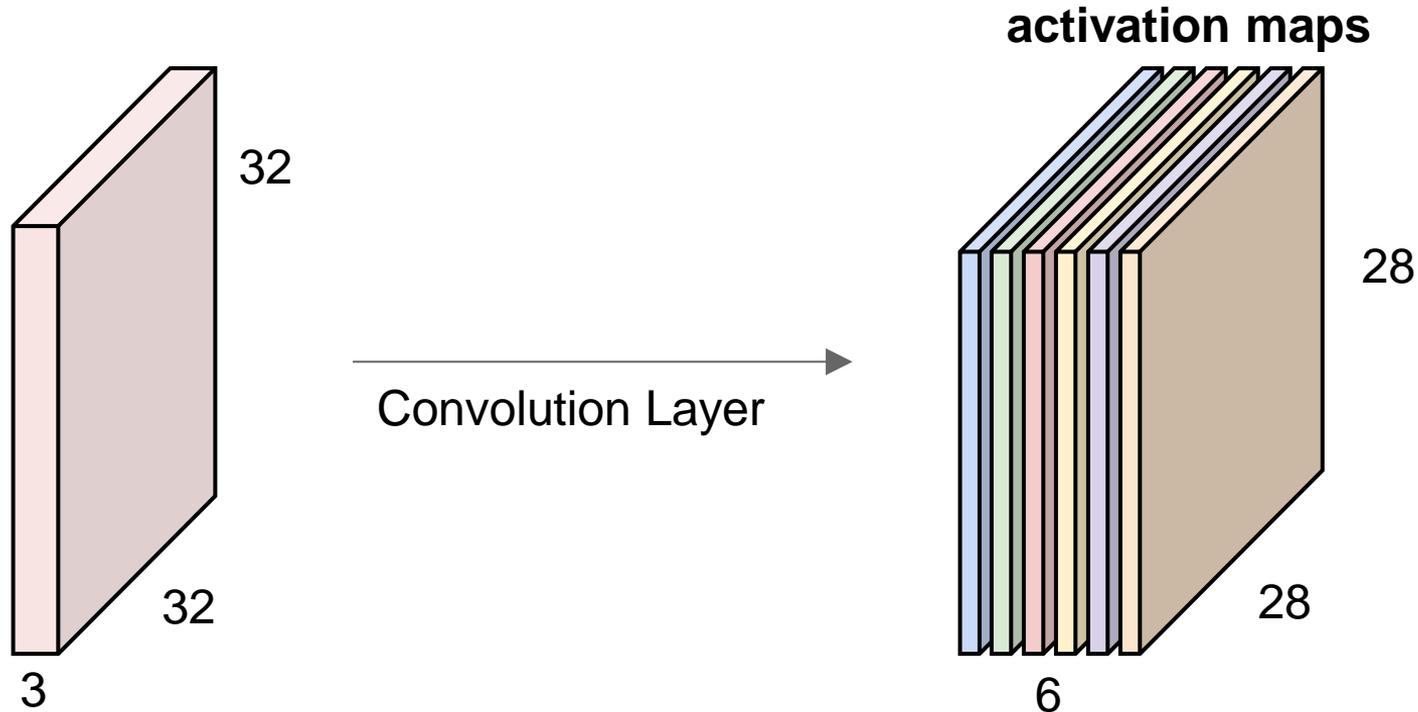


Convolution Layer

consider a second, **green** filter

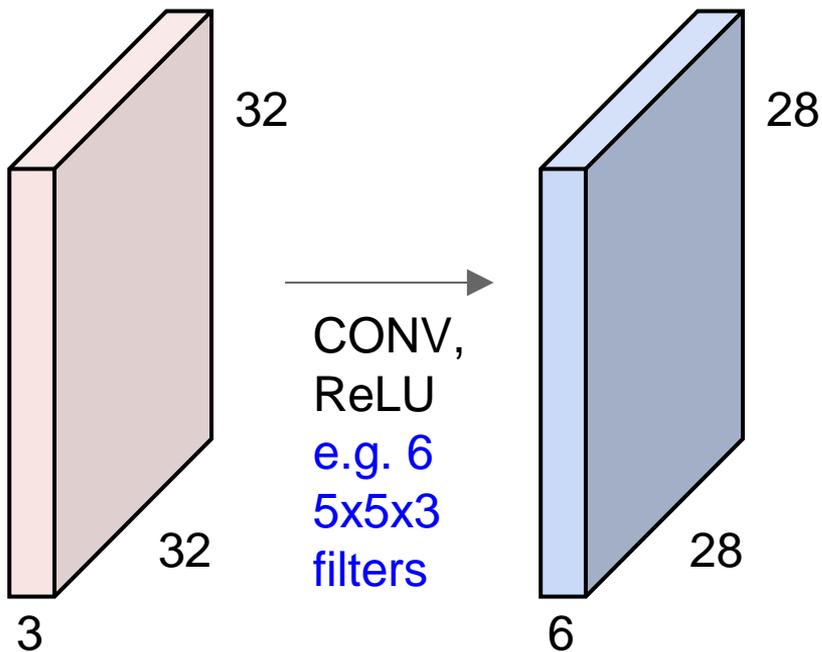


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

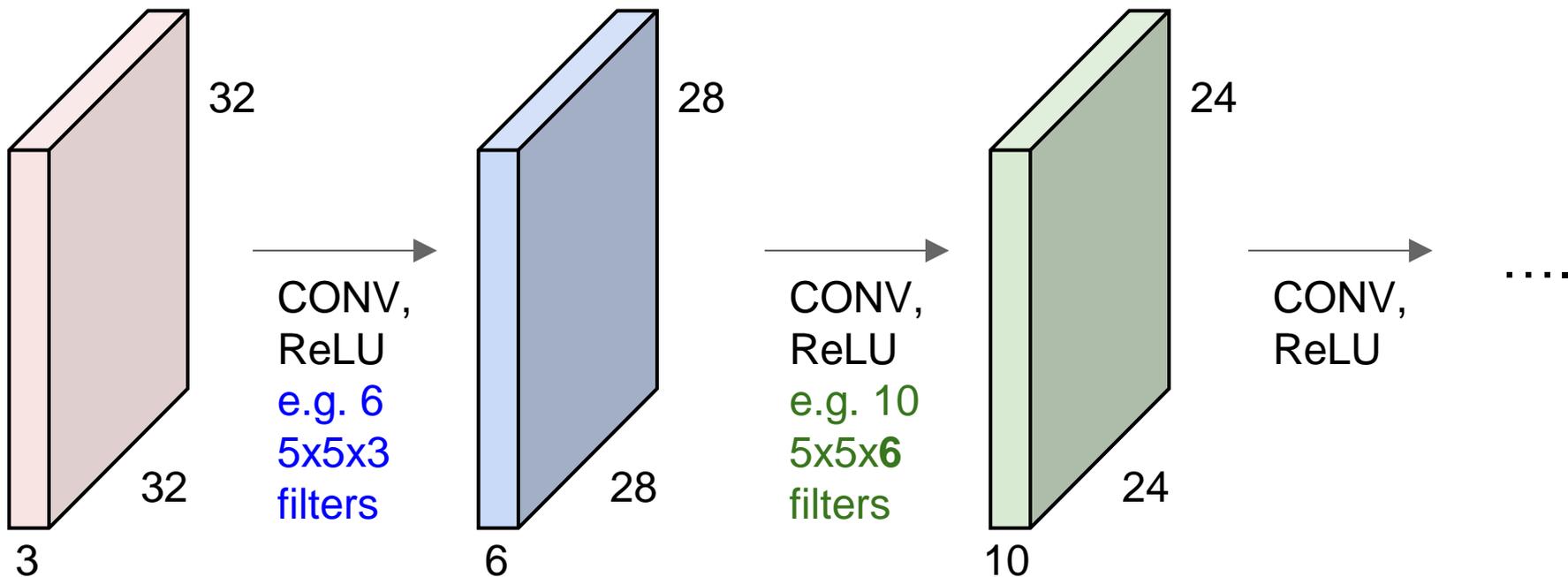


We stack these up to get a “new image” of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions

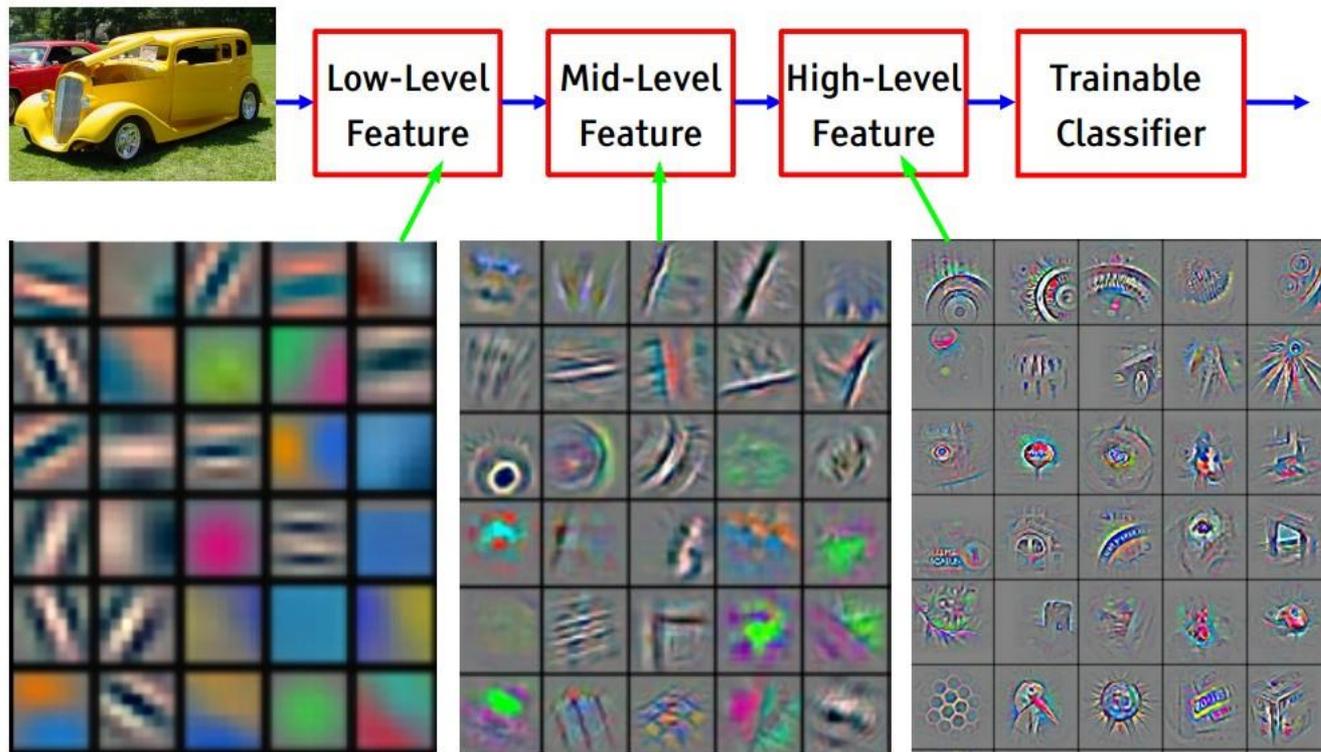


Preview: ConvNet is a sequence of Convolutional Layers, interspersed with activation functions



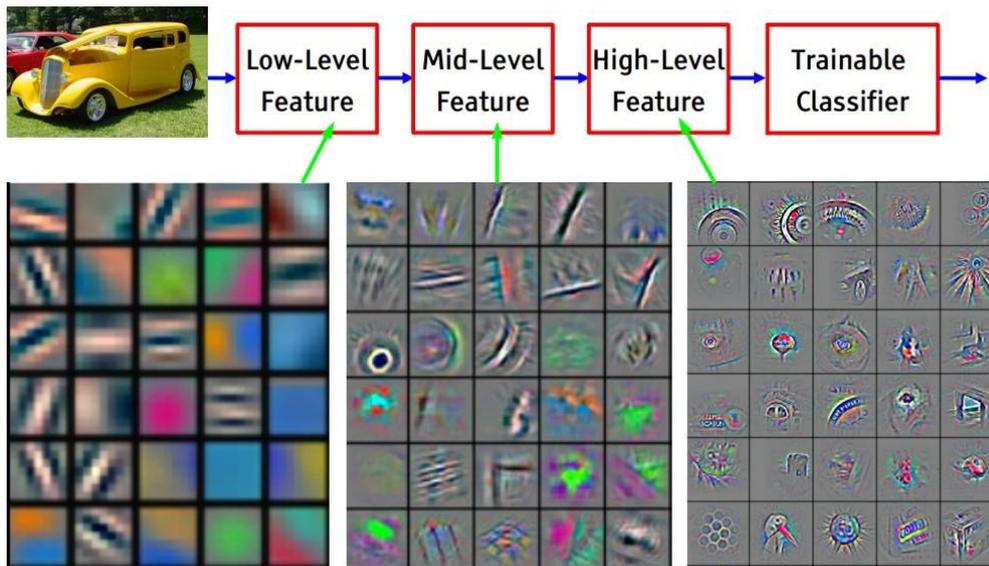
Preview

[From recent Yann
LeCun slides]



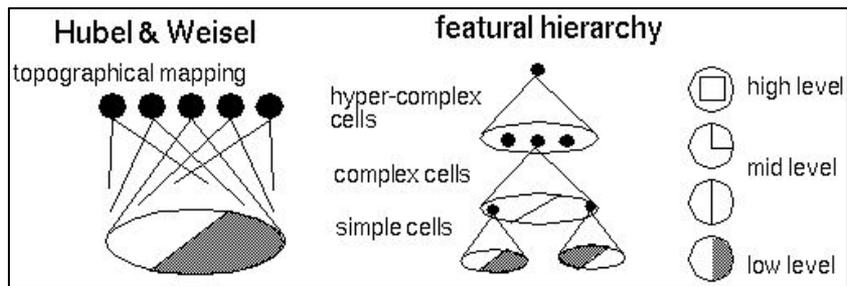
Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

Preview



[From recent Yann LeCun slides]

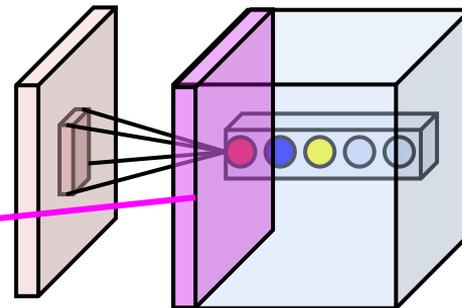
Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]



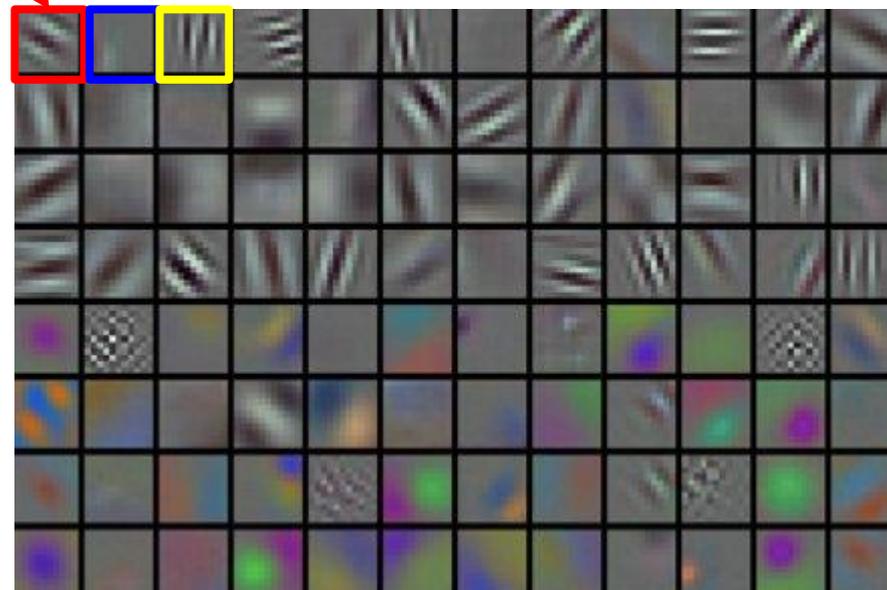
Activations:



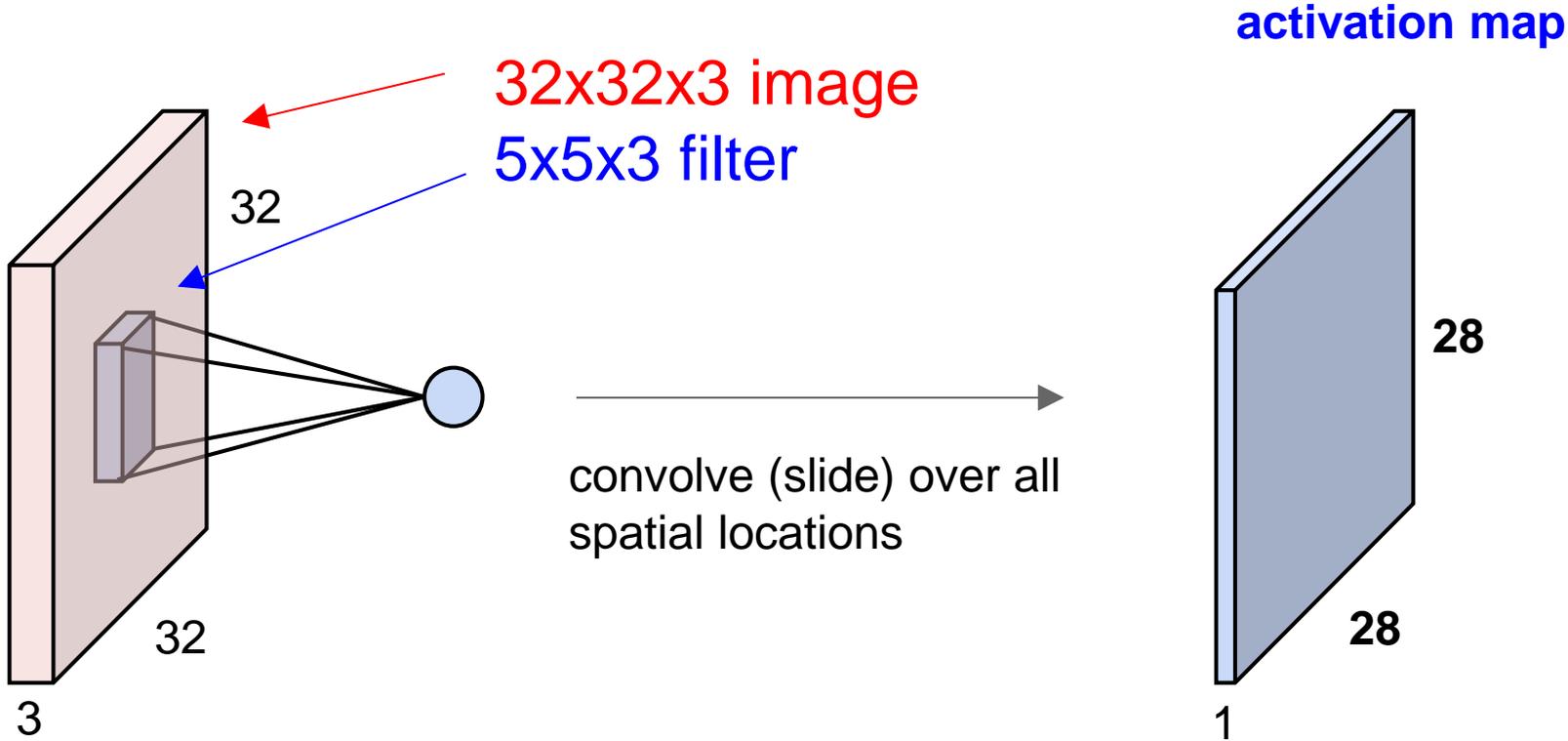
convolving the first filter in the input gives the first slice of depth in output volume



Activations:

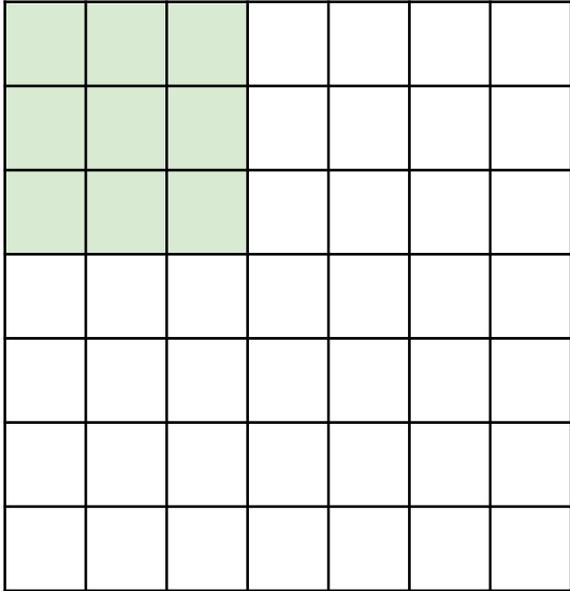


A closer look at spatial dimensions:



A closer look at spatial dimensions:

7

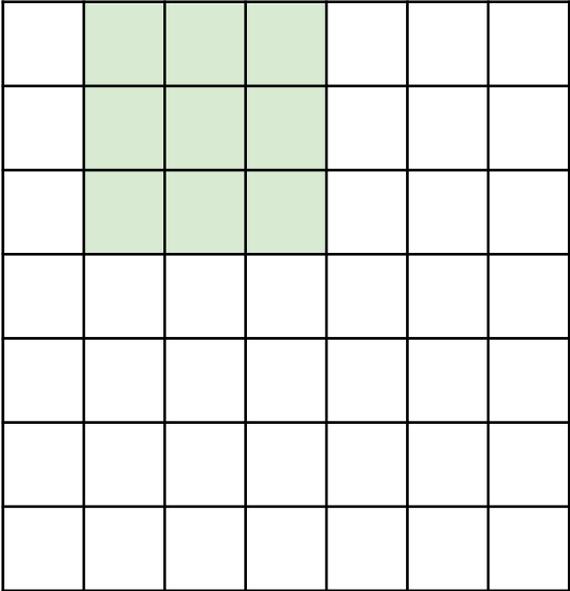


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

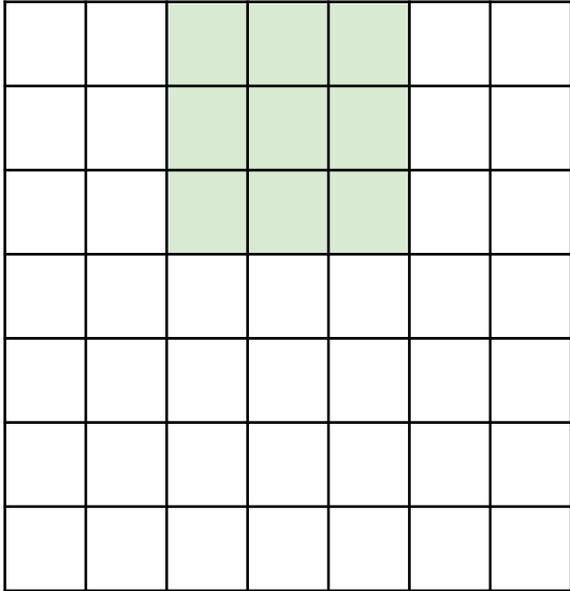


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

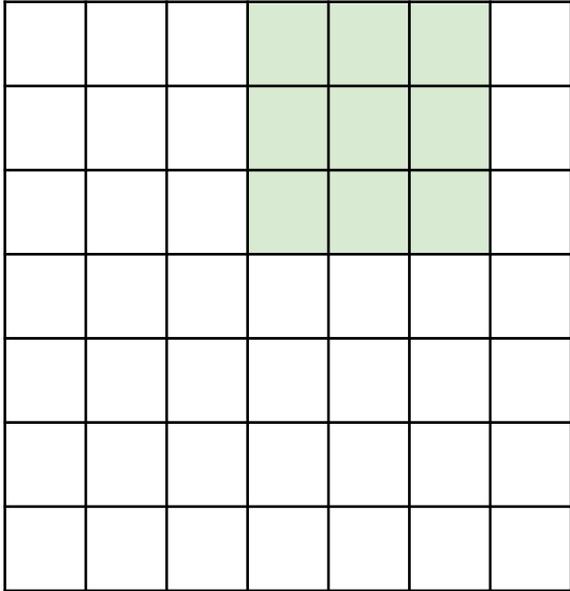


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

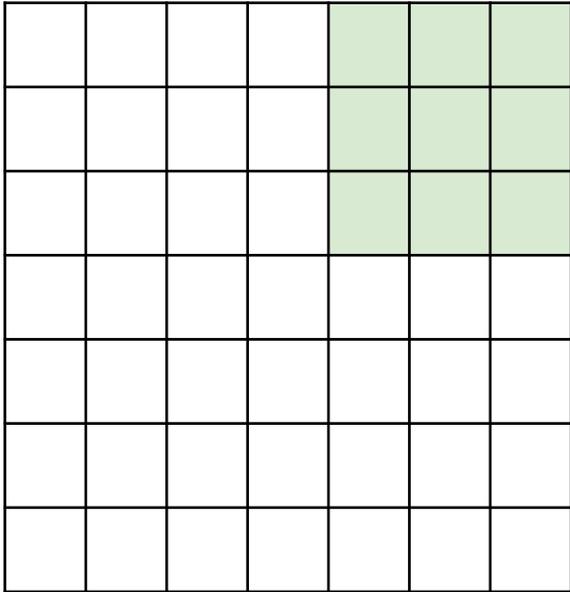


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7



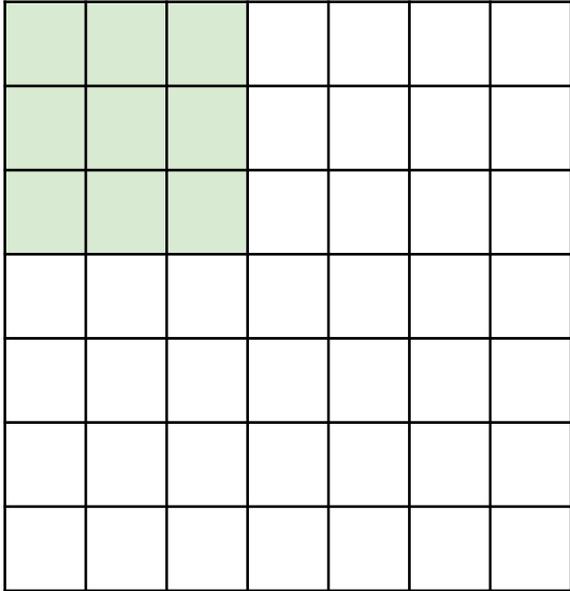
7x7 input (spatially)
assume 3x3 filter

=> 5x5 output

7

A closer look at spatial dimensions:

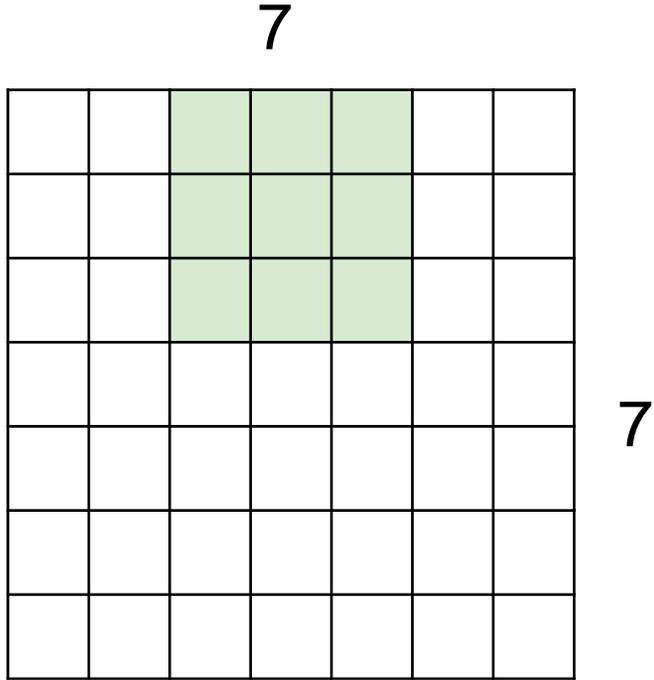
7



7

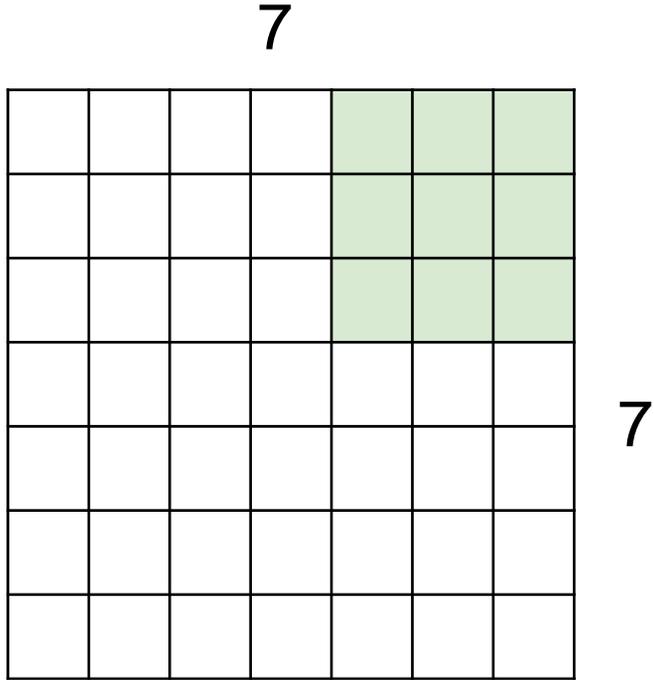
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

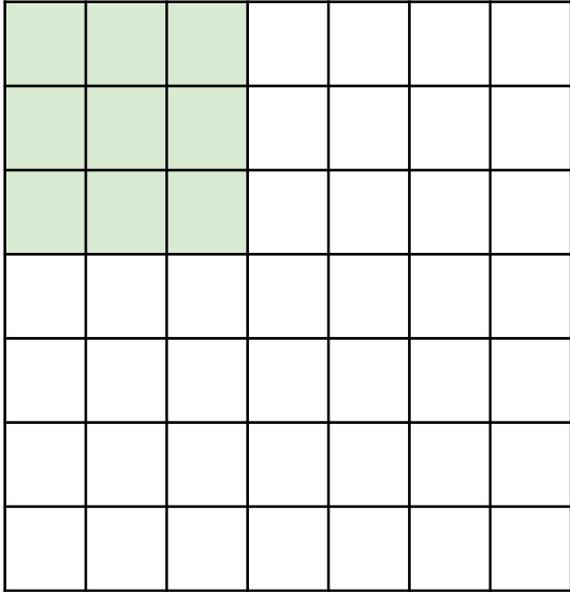
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:

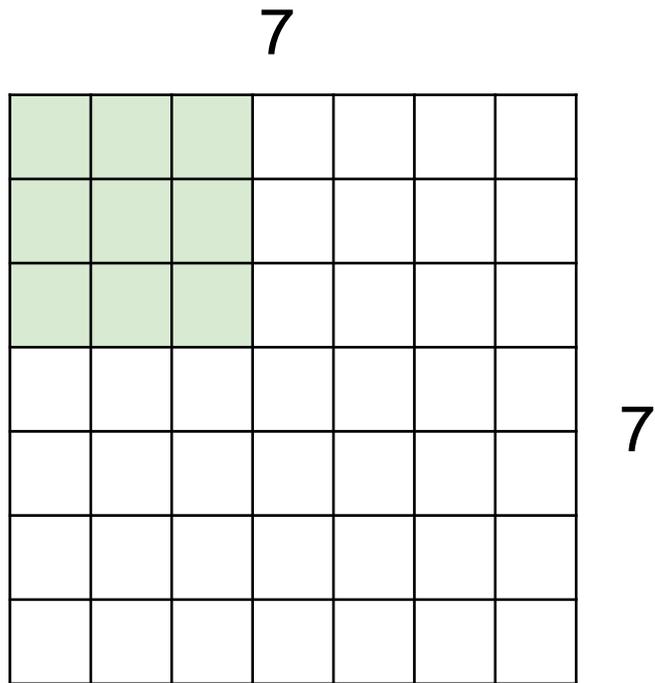
7



7

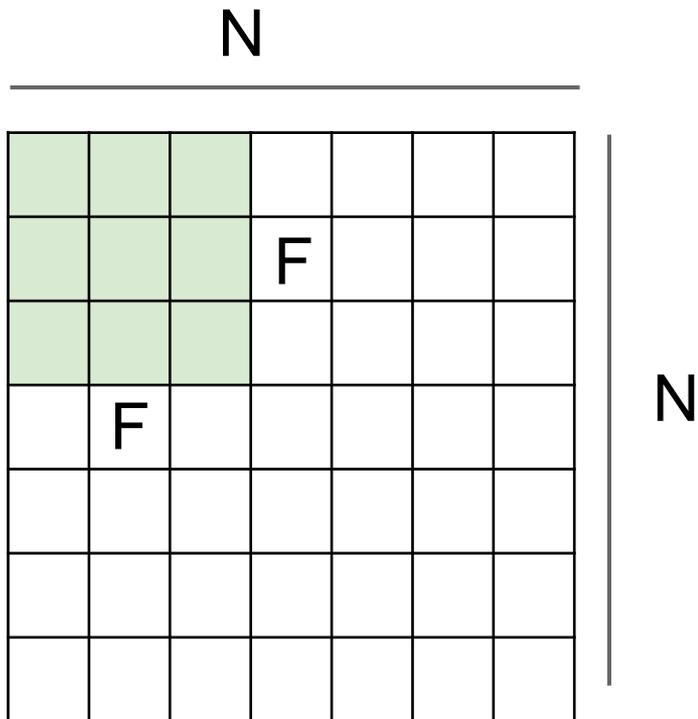
7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

doesn't fit!
cannot apply 3x3 filter on
7x7 input with stride 3.



Output size:
 $(N - F) / \text{stride} + 1$

e.g. $N = 7, F = 3$:

stride 1 $\Rightarrow (7 - 3) / 1 + 1 = 5$

stride 2 $\Rightarrow (7 - 3) / 2 + 1 = 3$

stride 3 $\Rightarrow (7 - 3) / 3 + 1 = 2.33 \text{ :}\backslash$

In practice: Common to zero pad the border

0	0	0	0	0	0			
0								
0								
0								
0								

e.g. input 7x7

3x3 filter, applied with **stride 1**

pad with 1 pixel border => what is the output?

(recall:)

$$(N - F) / \text{stride} + 1$$

In practice: Common to zero pad the border

0	0	0	0	0	0			
0								
0								
0								
0								

e.g. input 7x7

3x3 filter, applied with **stride 1**

pad with 1 pixel border => what is the output?

7x7 output!

In practice: Common to zero pad the border

0	0	0	0	0	0			
0								
0								
0								
0								

e.g. input 7x7

3x3 filter, applied with **stride 1**

pad with 1 pixel border => what is the output?

7x7 output!

in general, common to see CONV layers with stride 1, filters of size $F \times F$, and zero-padding with $(F-1)/2$. (will preserve size spatially)

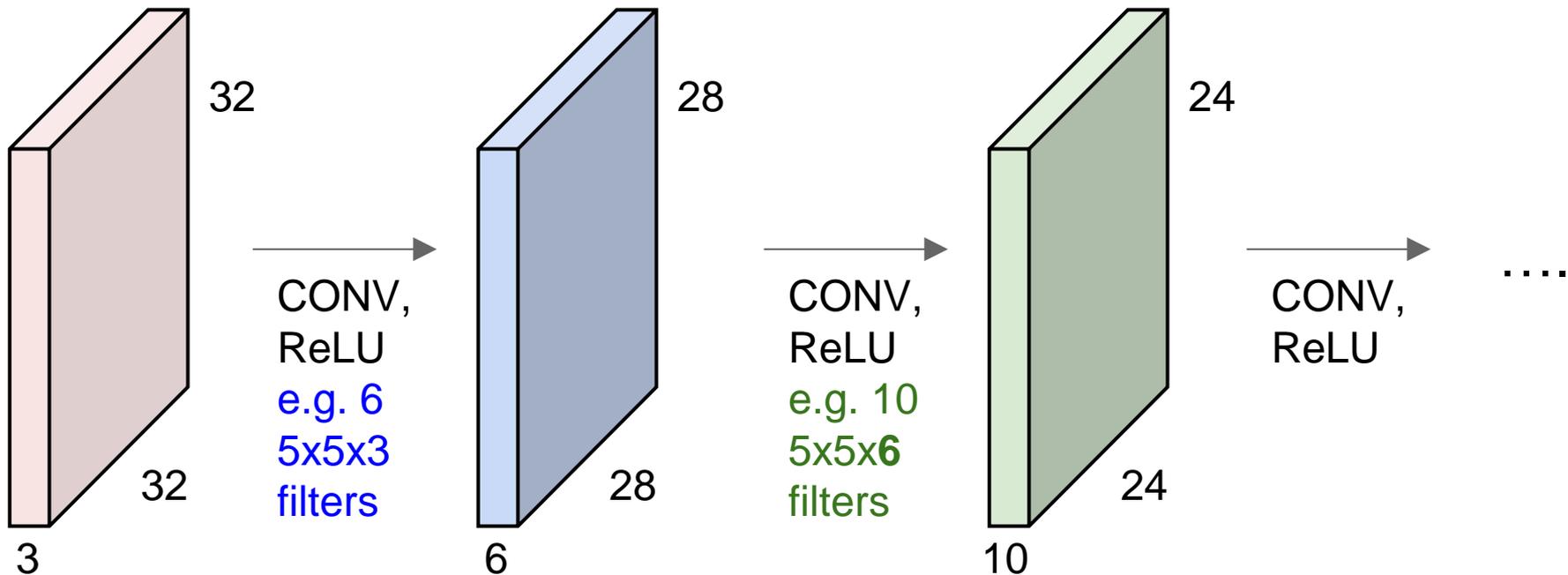
e.g. $F = 3 \Rightarrow$ zero pad with 1

$F = 5 \Rightarrow$ zero pad with 2

$F = 7 \Rightarrow$ zero pad with 3

Remember back to...

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.

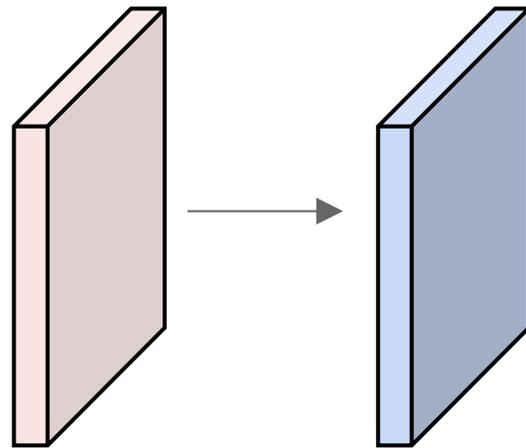


Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Output volume size: ?



Examples time:

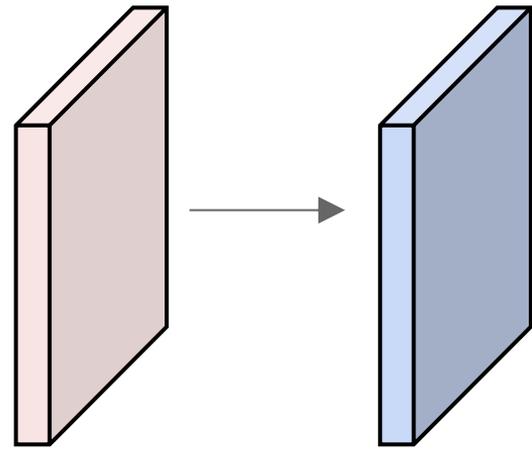
Input volume: **32x32x3**

10 **5x5** filters with stride **1**, pad **2**

Output volume size:

$(32+2*2-5)/1+1 = 32$ spatially, so

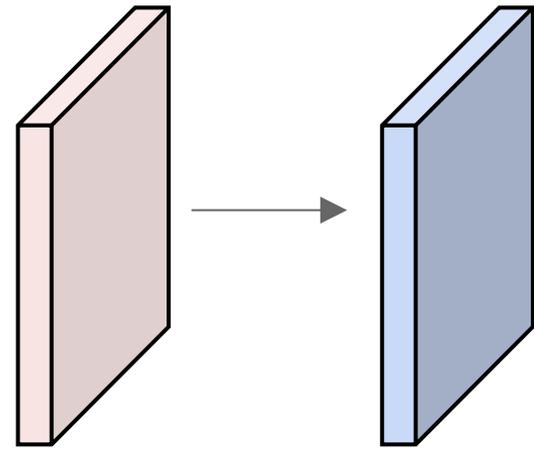
32x32x10



Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

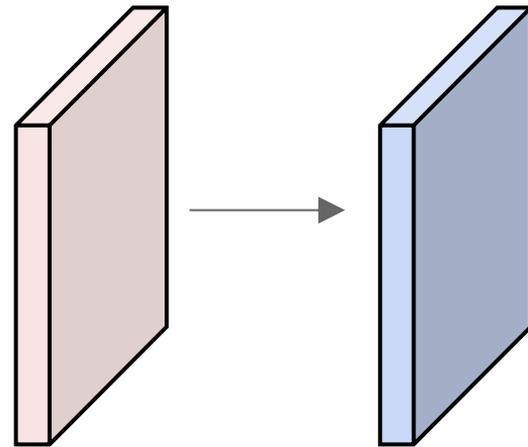


Number of parameters in this layer?

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?

each filter has $5*5*3 + 1 = 76$ params

(+1 for bias)

$\Rightarrow 76*10 = 760$

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
 - Number of filters K ,
 - their spatial extent F ,
 - the stride S ,
 - the amount of zero padding P .
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
 - $W_2 = (W_1 - F + 2P)/S + 1$
 - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
 - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d -th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d -th filter over the input volume with a stride of S , and then offset by d -th bias.

Common settings:

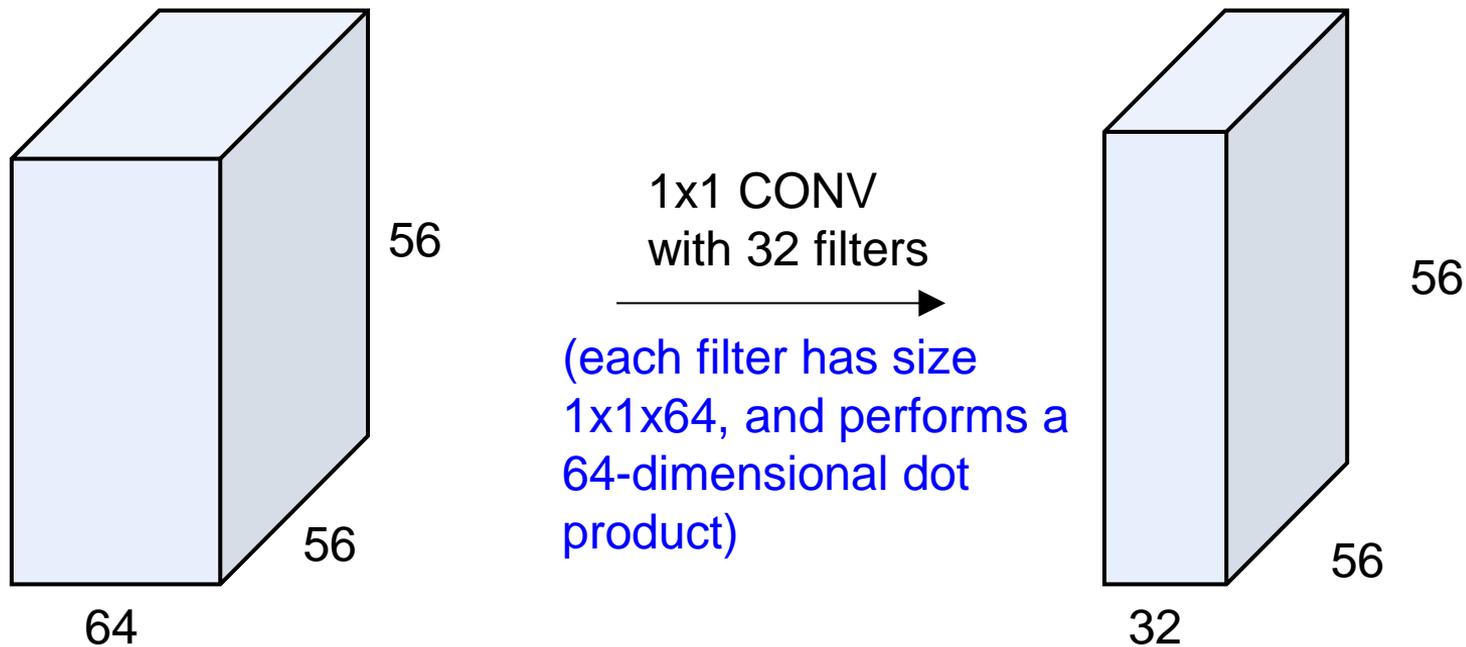
Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
 - Number of filters K ,
 - their spatial extent F ,
 - the stride S ,
 - the amount of zero padding P .
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
 - $W_2 = (W_1 - F + 2P)/S + 1$
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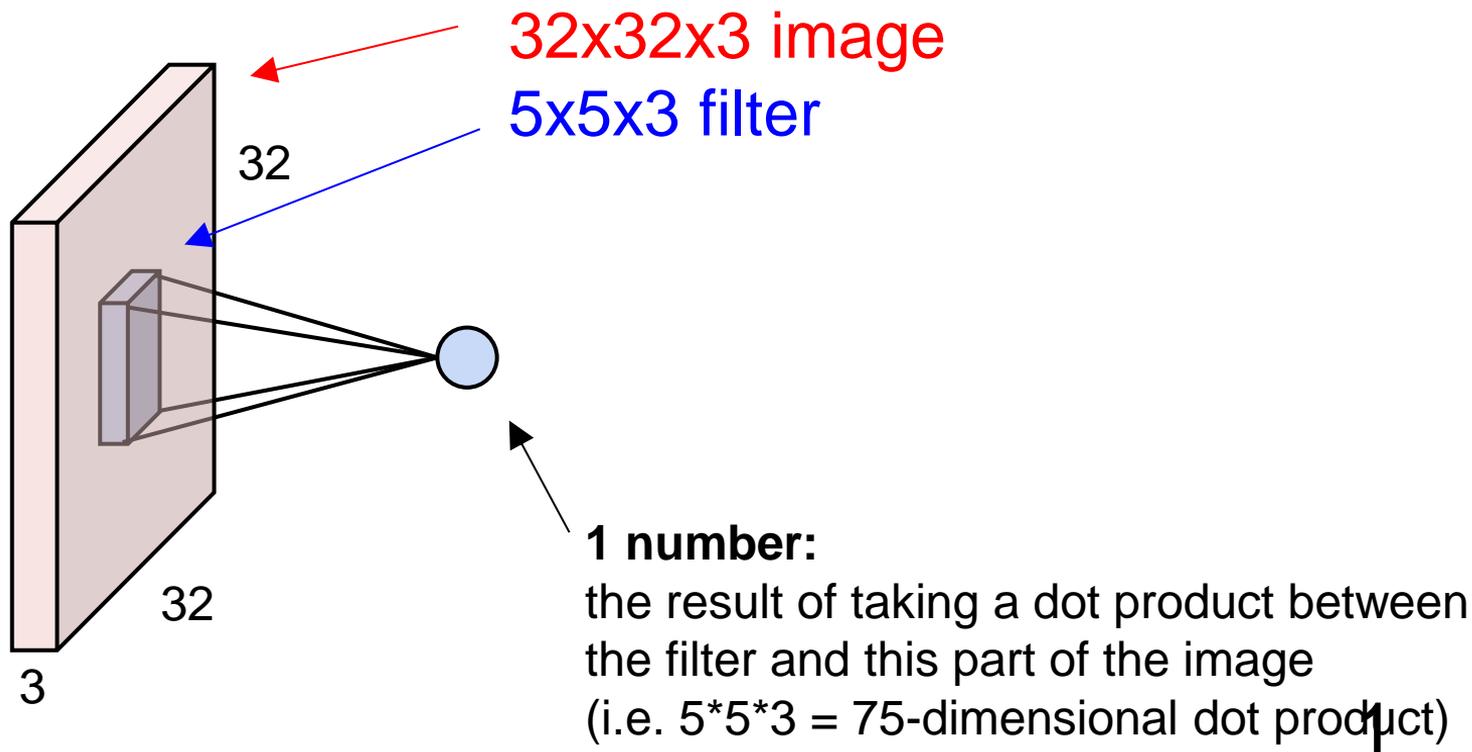
$K =$ (powers of 2, e.g. 32, 64, 128, 512)

- $F = 3, S = 1, P = 1$
- $F = 5, S = 1, P = 2$
- $F = 5, S = 2, P = ?$ (whatever fits)
- $F = 1, S = 1, P = 0$

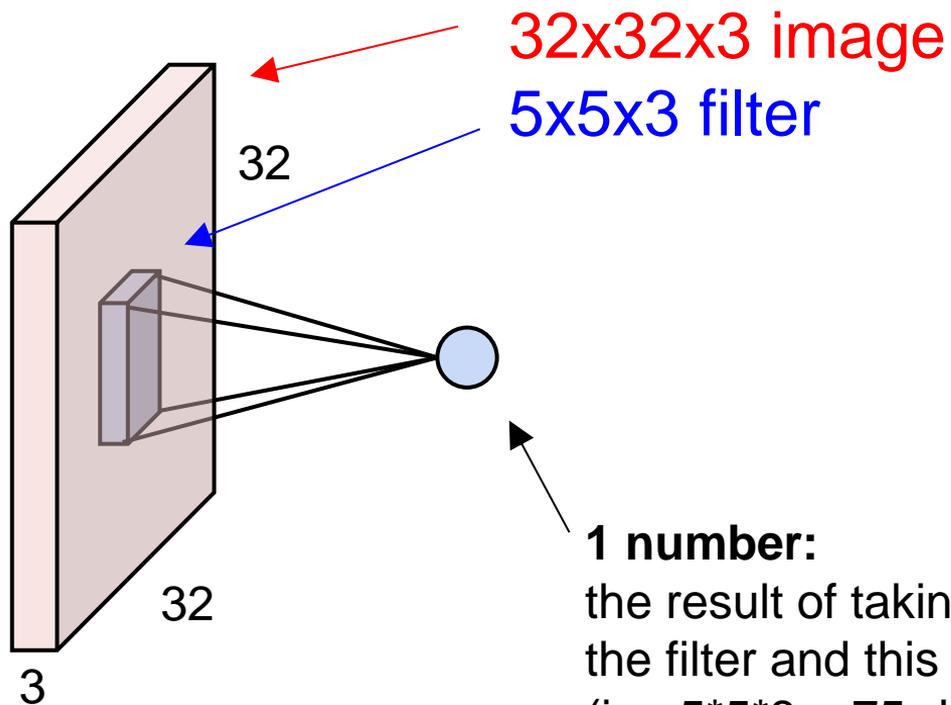
(btw, 1x1 convolution layers make perfect sense)



The brain/neuron view of CONV Layer

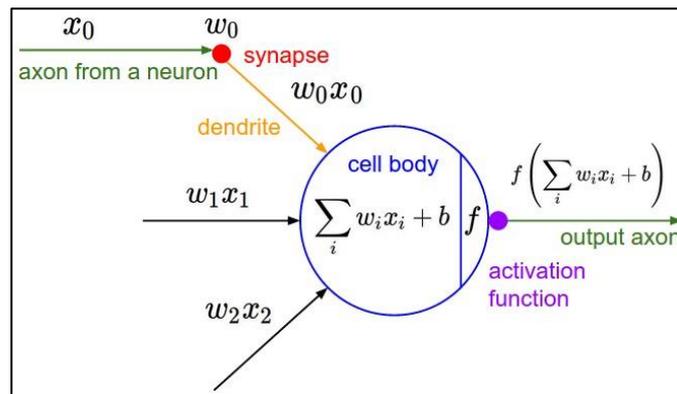


The brain/neuron view of CONV Layer



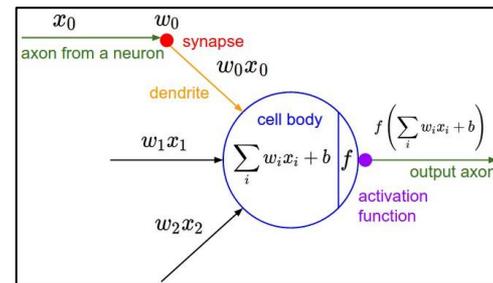
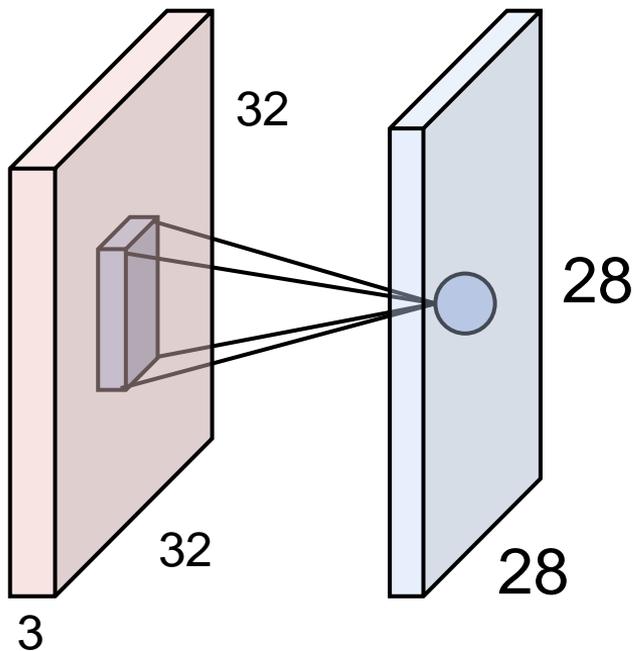
1 number:

the result of taking a dot product between the filter and this part of the image (i.e. $5*5*3 = 75$ -dimensional dot product)



It's just a neuron with local connectivity...

The brain/neuron view of CONV Layer

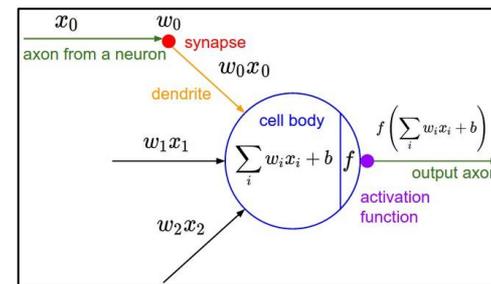
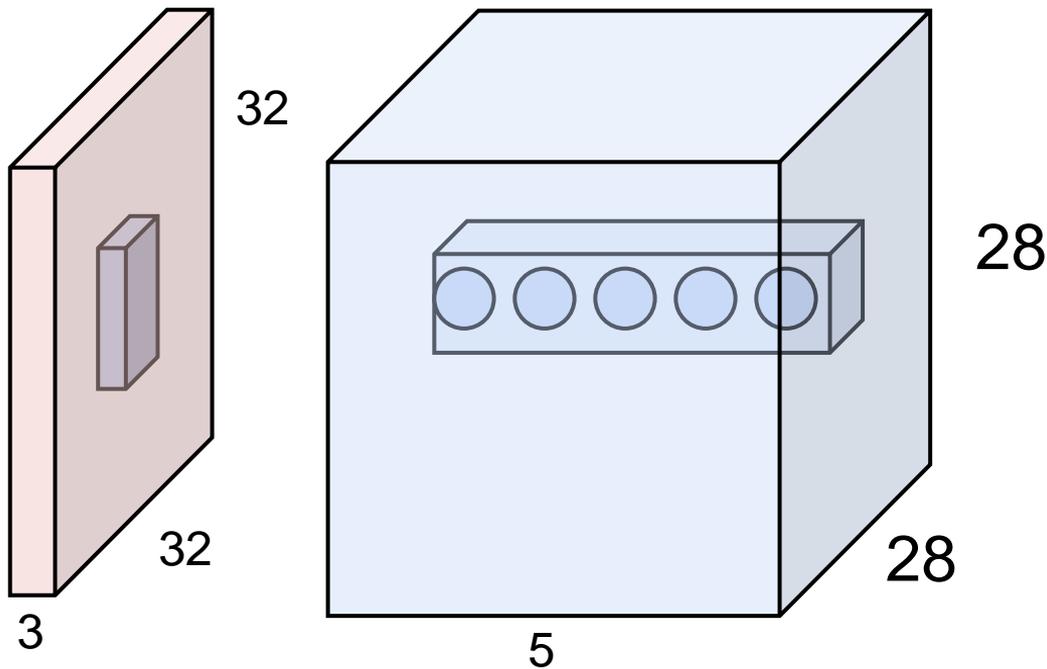


An activation map is a 28x28 sheet of neuron outputs:

1. Each is connected to a small region in the input
2. All of them share parameters

“5x5 filter” -> “5x5 receptive field for each neuron”

The brain/neuron view of CONV Layer



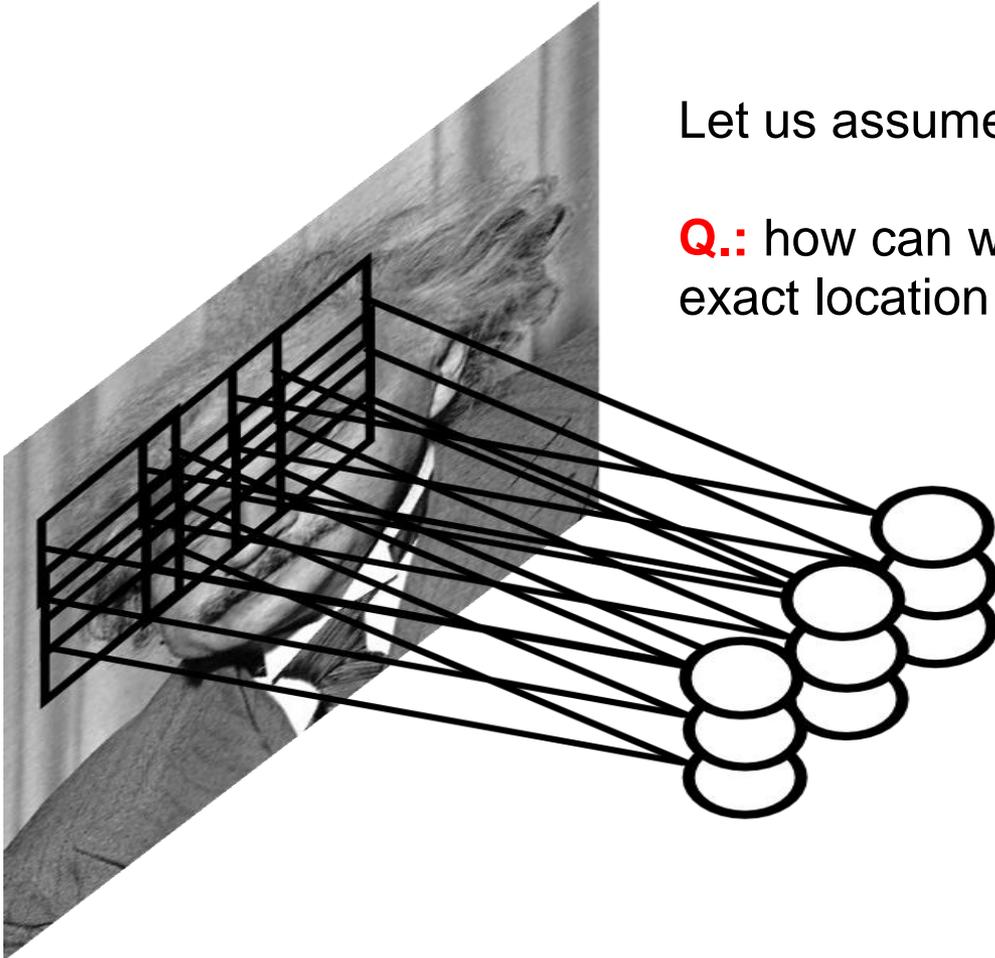
E.g. with 5 filters,
CONV layer consists of
neurons arranged in a 3D grid
(28x28x5)

There will be 5 different
neurons all looking at the same
region in the input volume

Pooling Layer

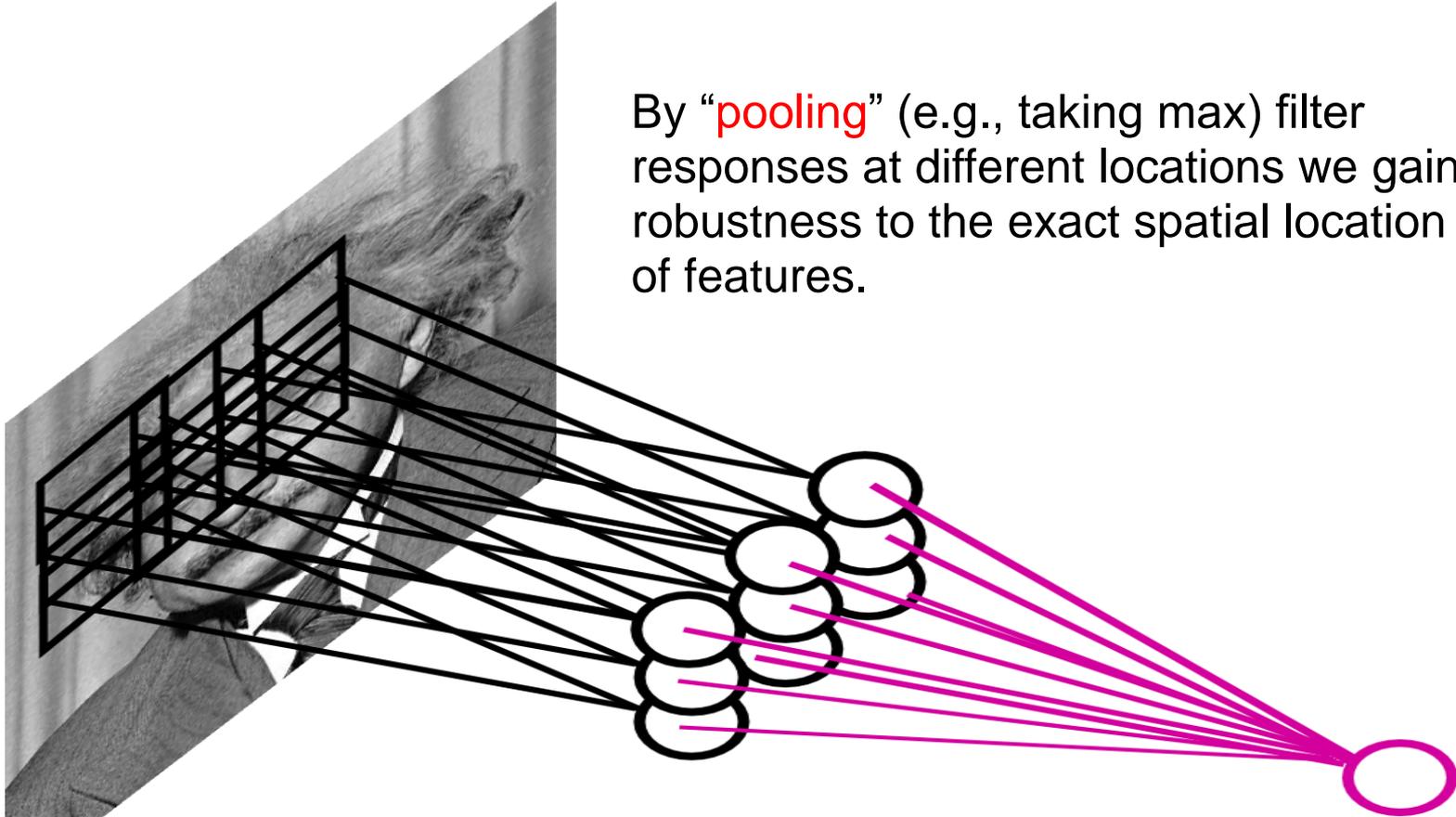
Let us assume filter is an “eye” detector.

Q.: how can we make the detection robust to the exact location of the eye?



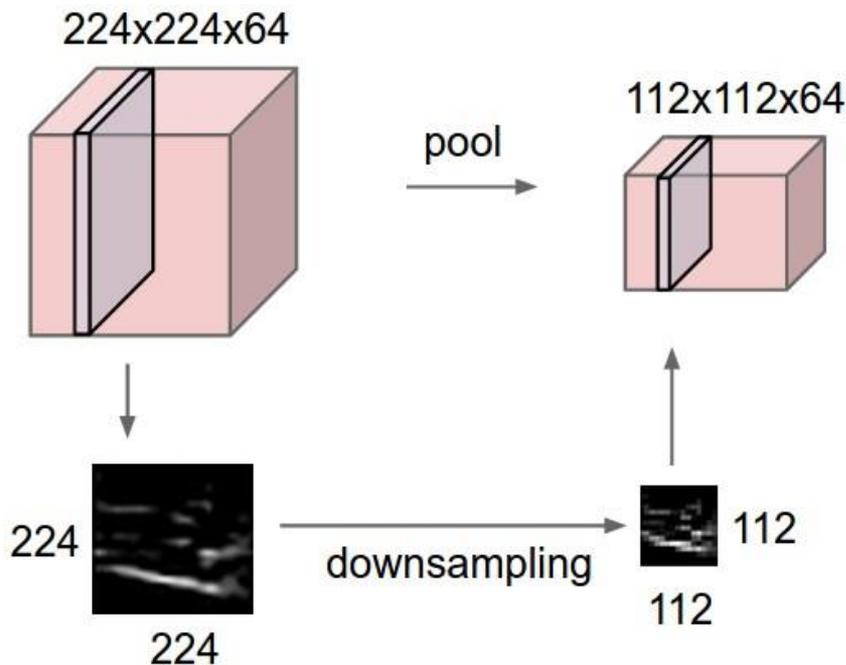
Pooling Layer

By “pooling” (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

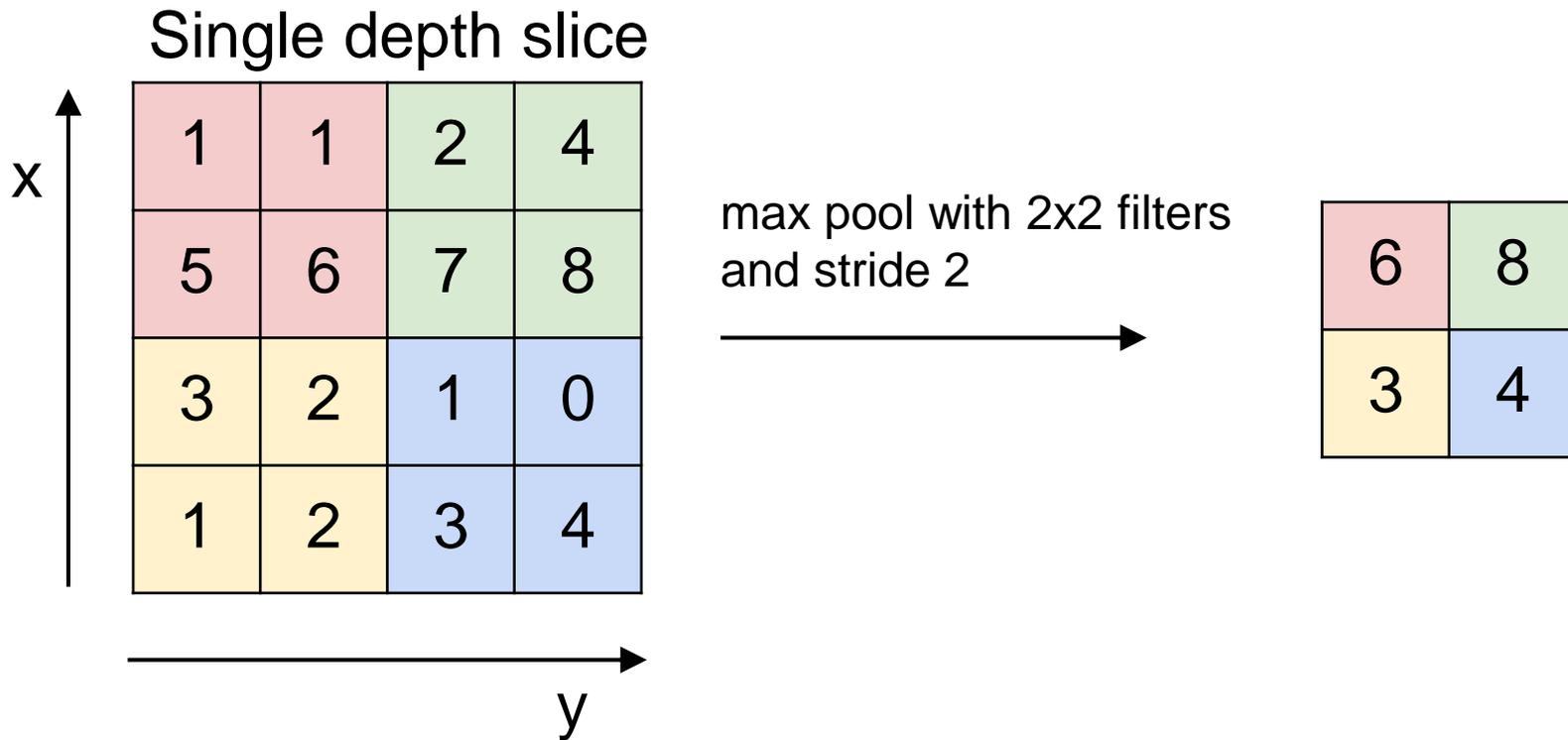


Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:

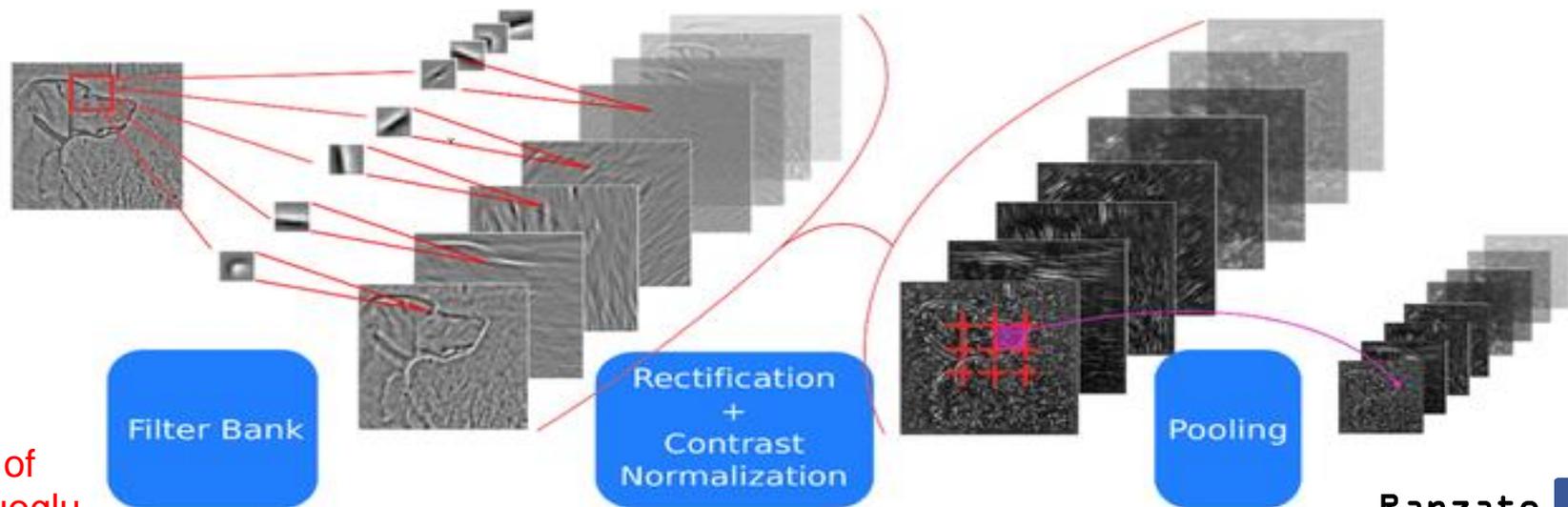
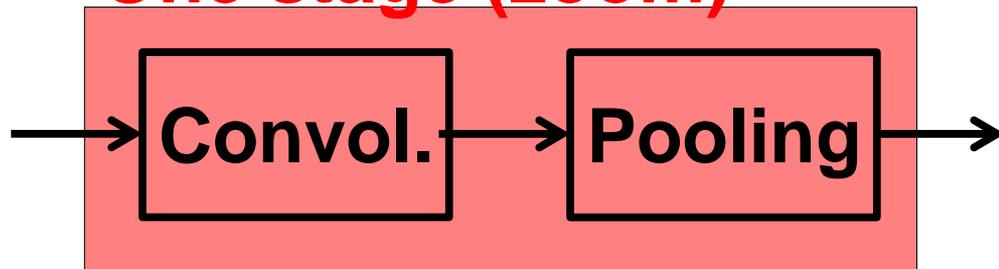


MAX POOLING



ConvNets: Typical Stage

One stage (zoom)



courtesy of
K. Kavukcuoglu

Ranzato

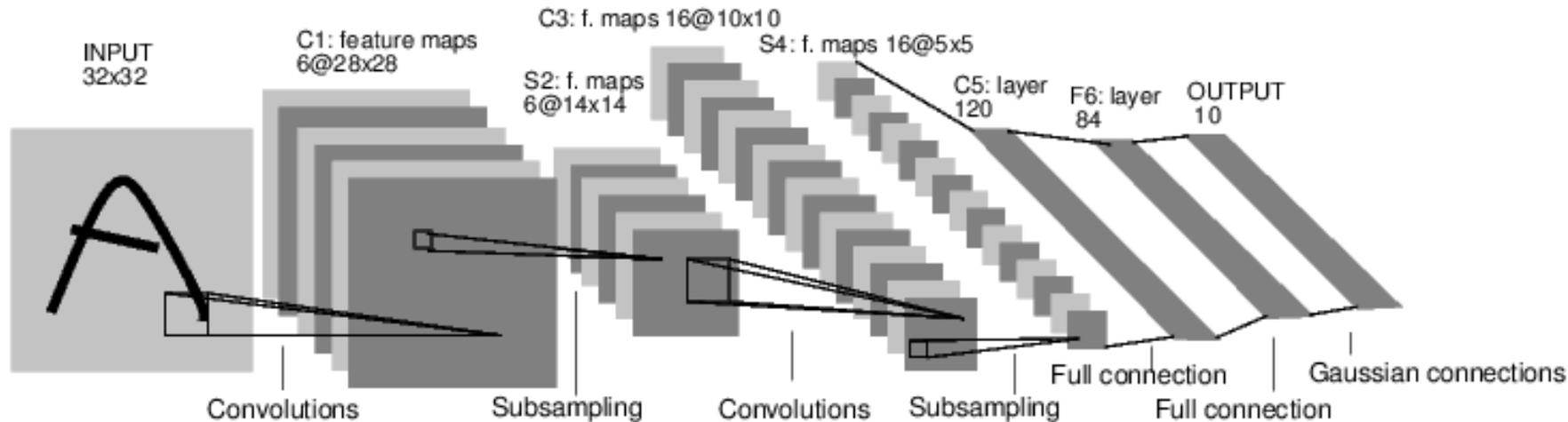


[ConvNetJS demo: training on CIFAR-10]

<http://cs.stanford.edu/people/karpathy/convnetjs/demo/cifar10.html>

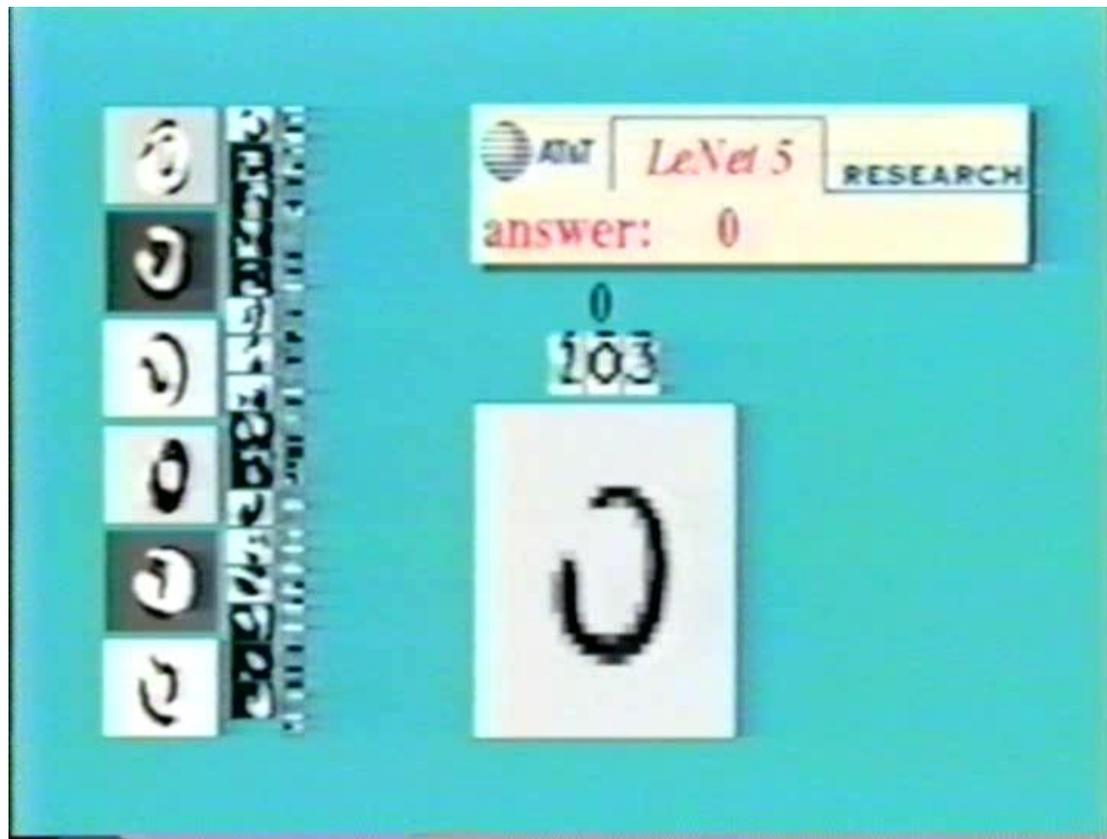
Case Study: LeNet-5

[LeCun et al., 1998]



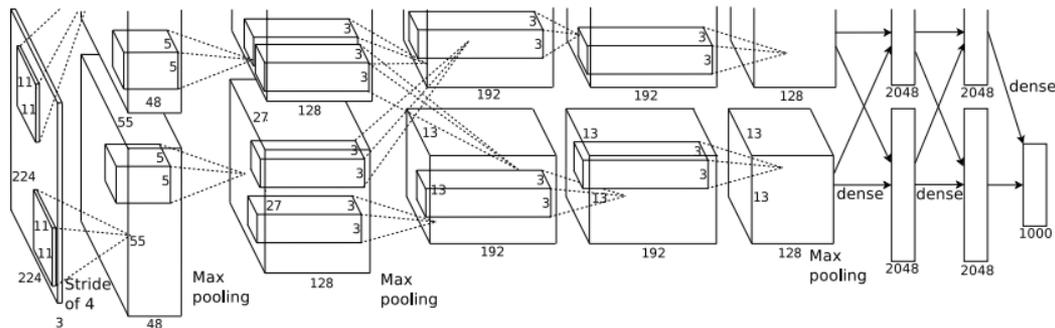
Conv filters were 5x5, applied at stride 1
Subsampling (Pooling) layers were 2x2 applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

Handwritten digit classification



Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

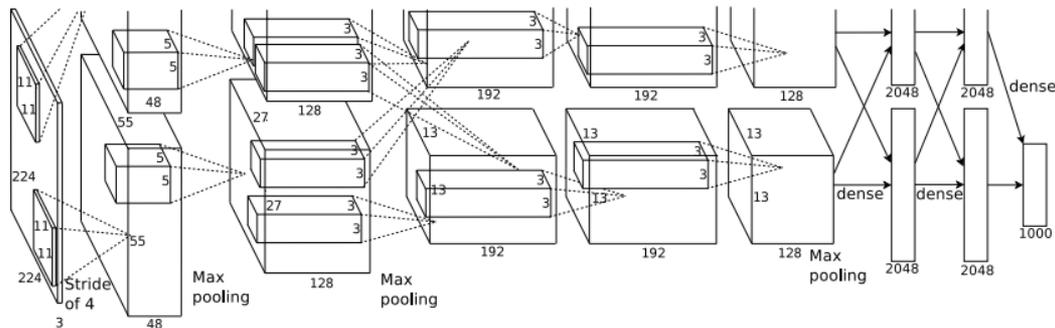
First layer (CONV1): 96 11x11 filters applied at stride 4

=>

Q: what is the output volume size? Hint: $(227-11)/4+1 = 55$

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4

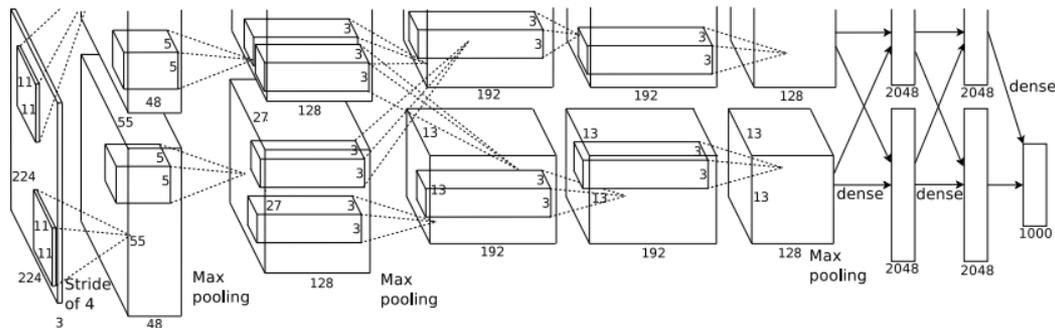
=>

Output volume **[55x55x96]**

Q: What is the total number of parameters in this layer?

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4

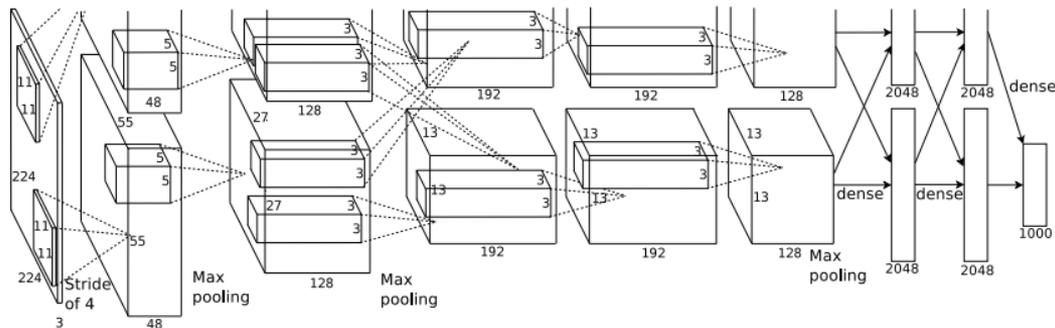
=>

Output volume **[55x55x96]**

Parameters: $(11*11*3)*96 = \mathbf{35K}$

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2

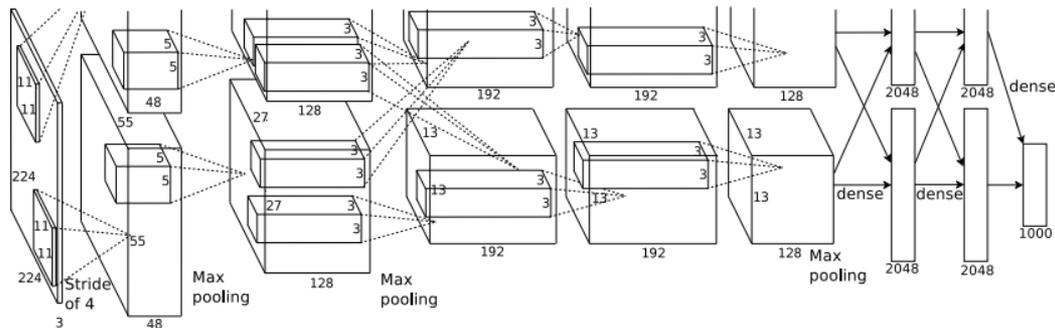
Output volume: 27x27x96

Q: what is the number of parameters in this layer?

1

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

After CONV1: 55x55x96

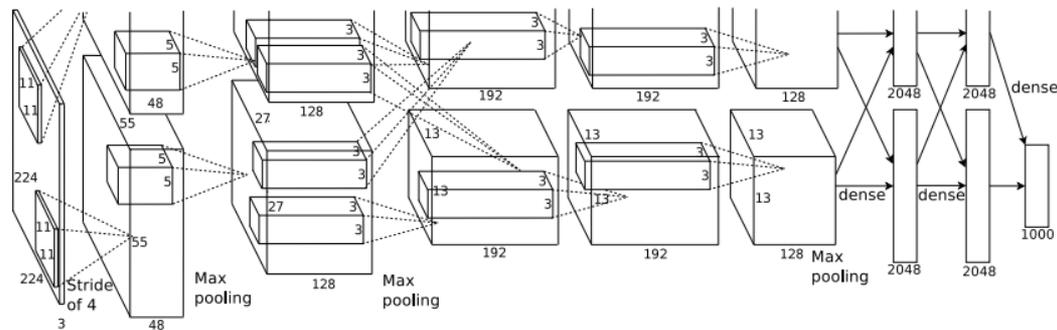
Second layer (POOL1): 3x3 filters applied at stride 2

Output volume: 27x27x96

Parameters: 0!

Case Study: AlexNet

[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:

[227x227x3] INPUT

[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0

[27x27x96] MAX POOL1: 3x3 filters at stride 2

[27x27x96] NORM1: Normalization layer

[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2

[13x13x256] MAX POOL2: 3x3 filters at stride 2

[13x13x256] NORM2: Normalization layer

[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1

[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1

[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1

[6x6x256] MAX POOL3: 3x3 filters at stride 2

[4096] FC6: 4096 neurons

[4096] FC7: 4096 neurons

[1000] FC8: 1000 neurons (class scores)

Case Study: AlexNet

[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture:

[227x227x3] INPUT

[55x55x96] **CONV1**: 96 11x11 filters at stride 4, pad 0

[27x27x96] **MAX POOL1**: 3x3 filters at stride 2

[27x27x96] **NORM1**: Normalization layer

[27x27x256] **CONV2**: 256 5x5 filters at stride 1, pad 2

[13x13x256] **MAX POOL2**: 3x3 filters at stride 2

[13x13x256] **NORM2**: Normalization layer

[13x13x384] **CONV3**: 384 3x3 filters at stride 1, pad 1

[13x13x384] **CONV4**: 384 3x3 filters at stride 1, pad 1

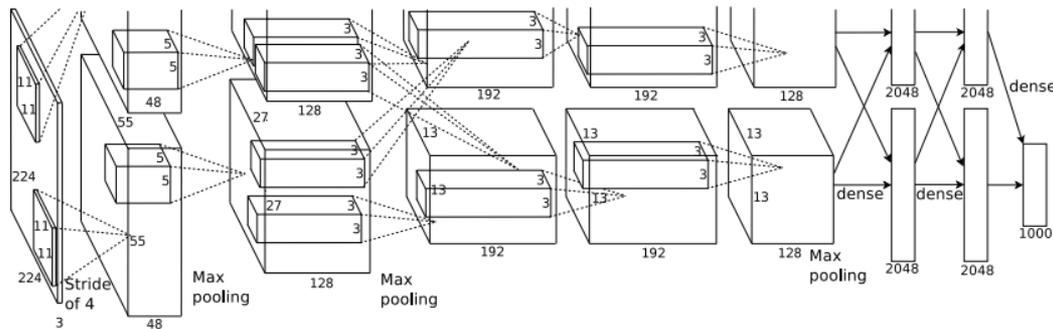
[13x13x256] **CONV5**: 256 3x3 filters at stride 1, pad 1

[6x6x256] **MAX POOL3**: 3x3 filters at stride 2

[4096] **FC6**: 4096 neurons

[4096] **FC7**: 4096 neurons

[1000] **FC8**: 1000 neurons (class scores)



Details/Retrospectives:

- first use of ReLU
- used Norm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%

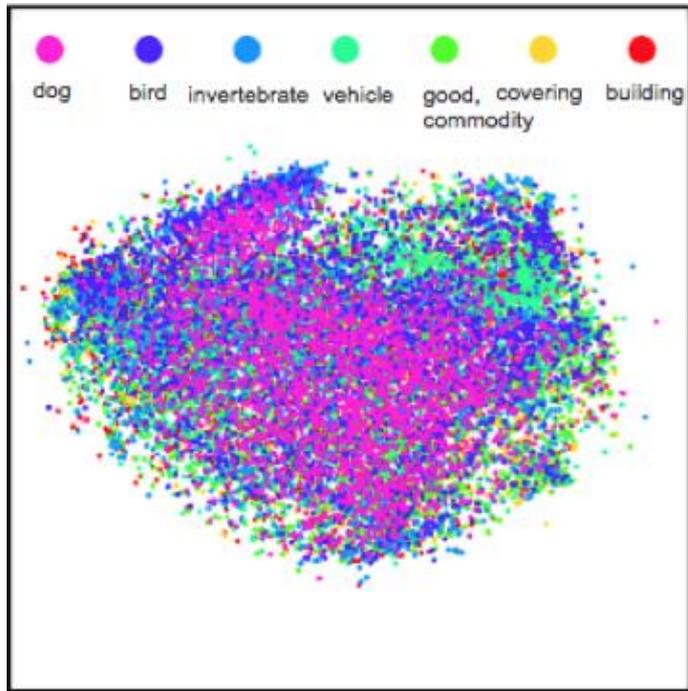
“You need a lot of a data if you want to train/use CNNs”

Transfer Learning

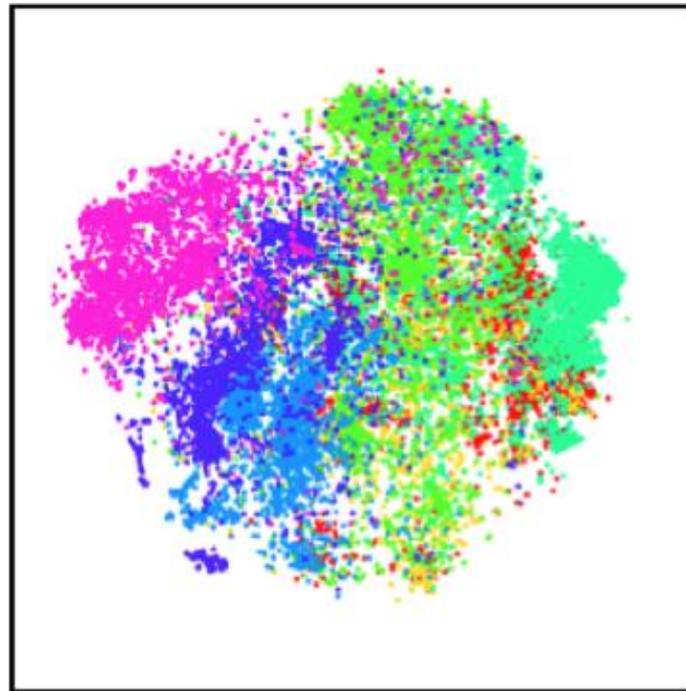
“You need a lot of data if you want to train the CNN”

NOT ALWAYS

The Unreasonable Effectiveness of Deep Features



Low-level: Pool₁



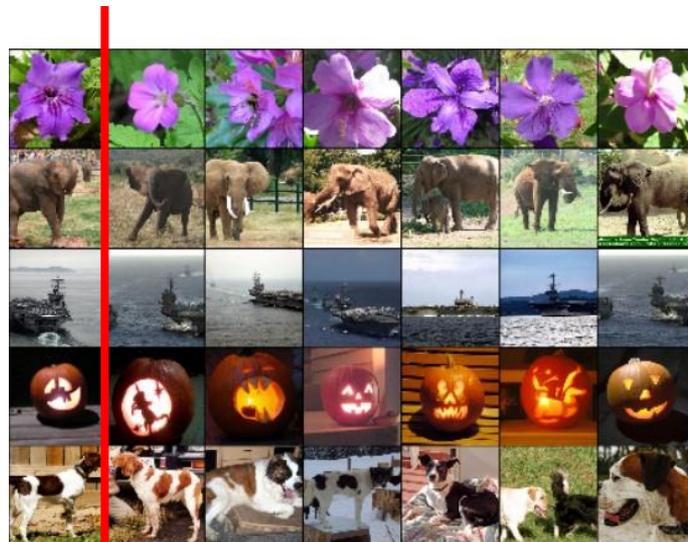
High-level: FC₆

Classes separate in the deep representations and transfer to many tasks.

[DeCAF] [Zeiler-Fergus]

Can be used as a generic feature

(“CNN code” = 4096-D vector before classifier)



query image

nearest neighbors in the “code” space

Transfer Learning with CNNs

image

conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

FC-4096

FC-4096

FC-1000

softmax

1. Train on Imagenet

Transfer Learning with CNNs



1. Train on Imagenet



2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end

Transfer Learning with CNNs

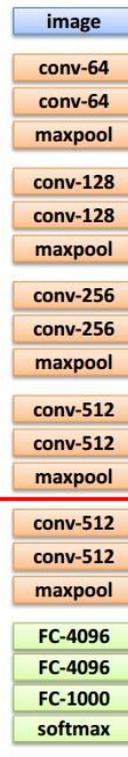


1. Train on ImageNet



2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

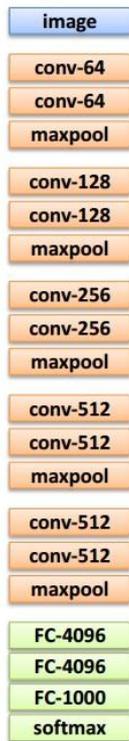
i.e. swap the Softmax layer at the end



3. If you have medium sized dataset, **“finetune”** instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.

Transfer Learning with CNNs



1. Train on ImageNet



2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end



3. If you have medium sized dataset, **“finetune”** instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.

tip: use only ~1/10th of the original learning rate in finetuning to player, and ~1/100th on intermediate layers

Case Study: VGGNet

[Simonyan and Zisserman, 2014]

Only 3x3 CONV stride 1, pad 1
and 2x2 MAX POOL stride 2

best model

11.2% top 5 error in ILSVRC 2013

->

7.3% top 5 error

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 LRN	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256	conv3-256	conv3-256	conv3-256 conv1-256	conv3-256 conv3-256	conv3-256 conv3-256 conv3-256
maxpool					
conv3-512	conv3-512	conv3-512	conv3-512 conv1-512	conv3-512 conv3-512	conv3-512 conv3-512 conv3-512
maxpool					
conv3-512	conv3-512	conv3-512	conv3-512 conv1-512	conv3-512 conv3-512	conv3-512 conv3-512 conv3-512
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Table 2: Number of parameters (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

INPUT: [224x224x3] memory: 224*224*3=150K params: 0 (not counting biases)

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864

POOL2: [112x112x64] memory: 112*112*64=800K params: 0

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456

POOL2: [56x56x128] memory: 56*56*128=400K params: 0

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

POOL2: [28x28x256] memory: 28*28*256=200K params: 0

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

POOL2: [14x14x512] memory: 14*14*512=100K params: 0

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

POOL2: [7x7x512] memory: 7*7*512=25K params: 0

FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448

FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216

FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

ConvNet Configuration			
B	C	D	
13 weight layers	16 weight layers	16 weight layers	19
Input (224 × 224 RGB image)			
conv3-64	conv3-64	conv3-64	conv3-64
conv3-64	conv3-64	conv3-64	conv3-64
maxpool			
conv3-128	conv3-128	conv3-128	conv3-128
conv3-128	conv3-128	conv3-128	conv3-128
maxpool			
conv3-256	conv3-256	conv3-256	conv3-256
conv3-256	conv3-256	conv3-256	conv3-256
	conv1-256	conv3-256	conv3-256
maxpool			
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
	conv1-512	conv3-512	conv3-512
maxpool			
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
	conv1-512	conv3-512	conv3-512
maxpool			
FC-4096			
FC-4096			
FC-1000			
soft-max			

INPUT: [224x224x3] memory: 224*224*3=150K params: 0 (not counting biases)

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864

POOL2: [112x112x64] memory: 112*112*64=800K params: 0

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456

POOL2: [56x56x128] memory: 56*56*128=400K params: 0

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

POOL2: [28x28x256] memory: 28*28*256=200K params: 0

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

POOL2: [14x14x512] memory: 14*14*512=100K params: 0

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

POOL2: [7x7x512] memory: 7*7*512=25K params: 0

FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448

FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216

FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~ = 93MB / image (only forward! ~*2 for bwd)

TOTAL params: 138M parameters

ConvNet Configuration			
B	C	D	
13 weight layers	16 weight layers	16 weight layers	19
Input (224 x 224 RGB image)			
conv3-64	conv3-64	conv3-64	conv3-64
conv3-64	conv3-64	conv3-64	conv3-64
maxpool			
conv3-128	conv3-128	conv3-128	conv3-128
conv3-128	conv3-128	conv3-128	conv3-128
maxpool			
conv3-256	conv3-256	conv3-256	conv3-256
conv3-256	conv3-256	conv3-256	conv3-256
	conv1-256	conv3-256	conv3-256
maxpool			
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
	conv1-512	conv3-512	conv3-512
maxpool			
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
	conv1-512	conv3-512	conv3-512
maxpool			
FC-4096			
FC-4096			
FC-1000			
soft-max			

INPUT: [224x224x3] memory: $224*224*3=150K$ params: 0 (not counting biases)

CONV3-64: [224x224x64] memory: $224*224*64=3.2M$ params: $(3*3*3)*64 = 1,728$

CONV3-64: [224x224x64] memory: $224*224*64=3.2M$ params: $(3*3*64)*64 = 36,864$

POOL2: [112x112x64] memory: $112*112*64=800K$ params: 0

CONV3-128: [112x112x128] memory: $112*112*128=1.6M$ params: $(3*3*64)*128 = 73,728$

CONV3-128: [112x112x128] memory: $112*112*128=1.6M$ params: $(3*3*128)*128 = 147,456$

POOL2: [56x56x128] memory: $56*56*128=400K$ params: 0

CONV3-256: [56x56x256] memory: $56*56*256=800K$ params: $(3*3*128)*256 = 294,912$

CONV3-256: [56x56x256] memory: $56*56*256=800K$ params: $(3*3*256)*256 = 589,824$

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POOL2: [28x28x256] memory: $28*28*256=200K$ params: 0

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POOL2: [14x14x512] memory: $14*14*512=100K$ params: 0

CONV3-512: [14x14x512] memory: $14*14*512=100K$ params: $(3*3*512)*512 = 2,359,296$

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POOL2: [7x7x512] memory: $7*7*512=25K$ params: 0

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FC: [1x1x4096] memory: 4096 params: $4096*4096 = 16,777,216$

FC: [1x1x1000] memory: 1000 params: $4096*1000 = 4,096,000$

TOTAL memory: $24M * 4 \text{ bytes} \approx 93MB / \text{image}$ (only forward! ~ 2 for bwd)

TOTAL params: 138M parameters

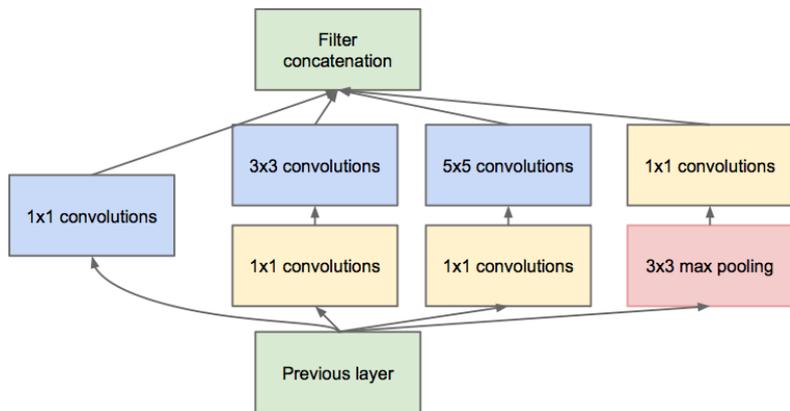
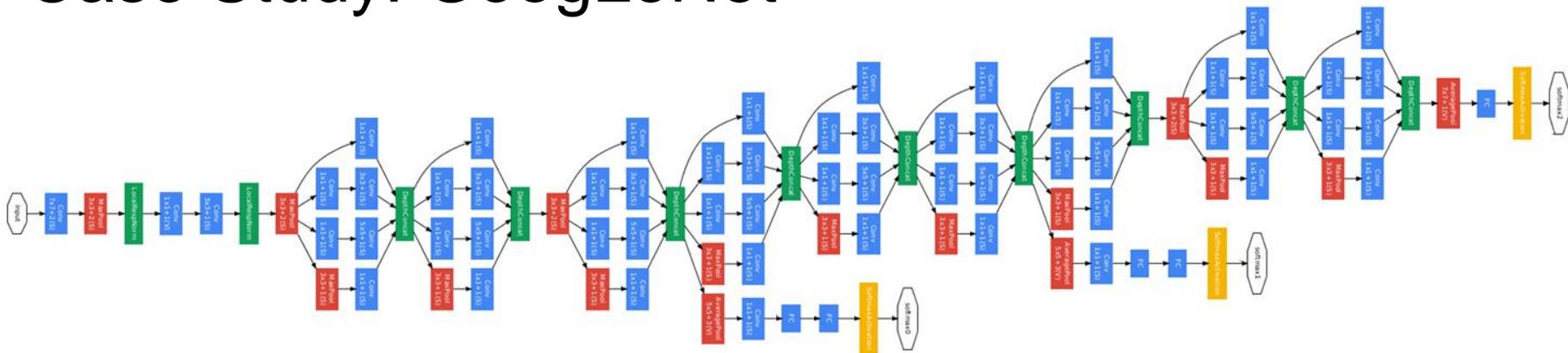
Note:

Most memory is in early CONV

Most params are in late FC

Case Study: GoogLeNet

[Szegedy et al., 2014]



Inception module

ILSVRC 2014 winner (6.7% top 5 error)

Case Study: GoogLeNet

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

Fun features:

- Only 5 million params!
(Removes FC layers completely)

Compared to AlexNet:

- 12X less params
- 2x more compute
- 6.67% (vs. 16.4%)

Case Study: ResNet [He et al., 2015]

ILSVRC 2015 winner (3.6% top 5 error)



MSRA @ ILSVRC & COCO 2015 Competitions

- **1st places in all five main tracks**
 - ImageNet Classification: “Ultra-deep” (quote Yann) **152-layer** nets
 - ImageNet Detection: **16%** better than 2nd
 - ImageNet Localization: **27%** better than 2nd
 - COCO Detection: **11%** better than 2nd
 - COCO Segmentation: **12%** better than 2nd

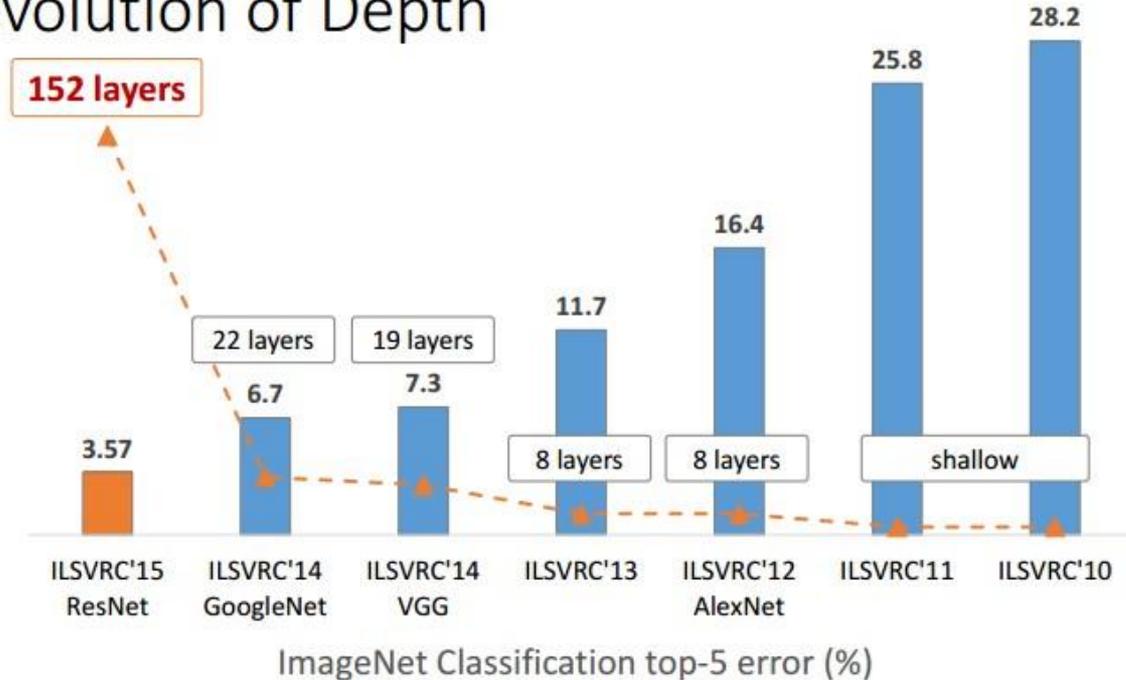
*improvements are relative numbers



Kaiming He, Xiangyu Zhang, Shaoqing Ren, & Jian Sun. “Deep Residual Learning for Image Recognition”. arXiv 2015.

Slide from Kaiming He’s recent presentation <https://www.youtube.com/watch?v=1PGLj-uKT1w>

Revolution of Depth



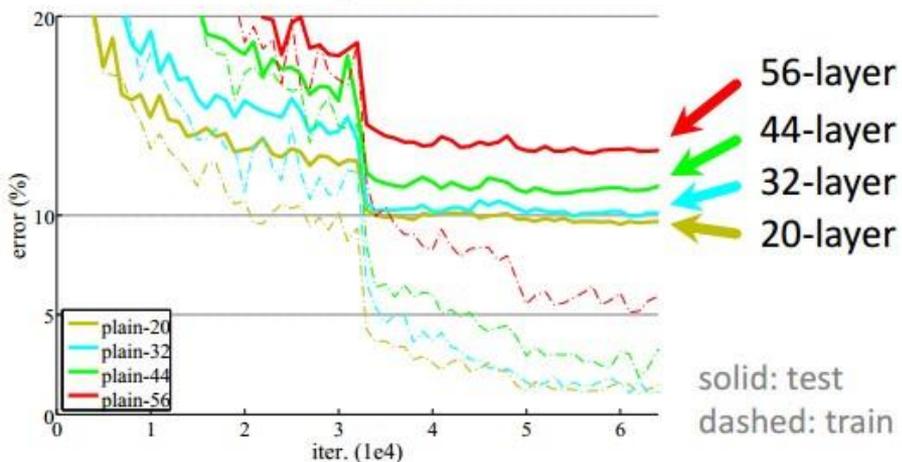
ImageNet Classification top-5 error (%)

Kaiming He, Xiangyu Zhang, Shaoqing Ren, & Jian Sun. "Deep Residual Learning for Image Recognition". arXiv 2015.

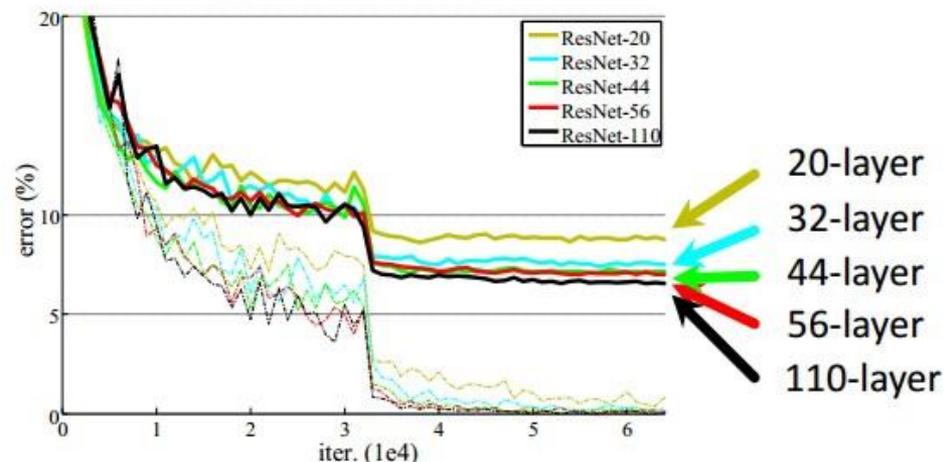
(slide from Kaiming He's recent presentation)

CIFAR-10 experiments

CIFAR-10 plain nets

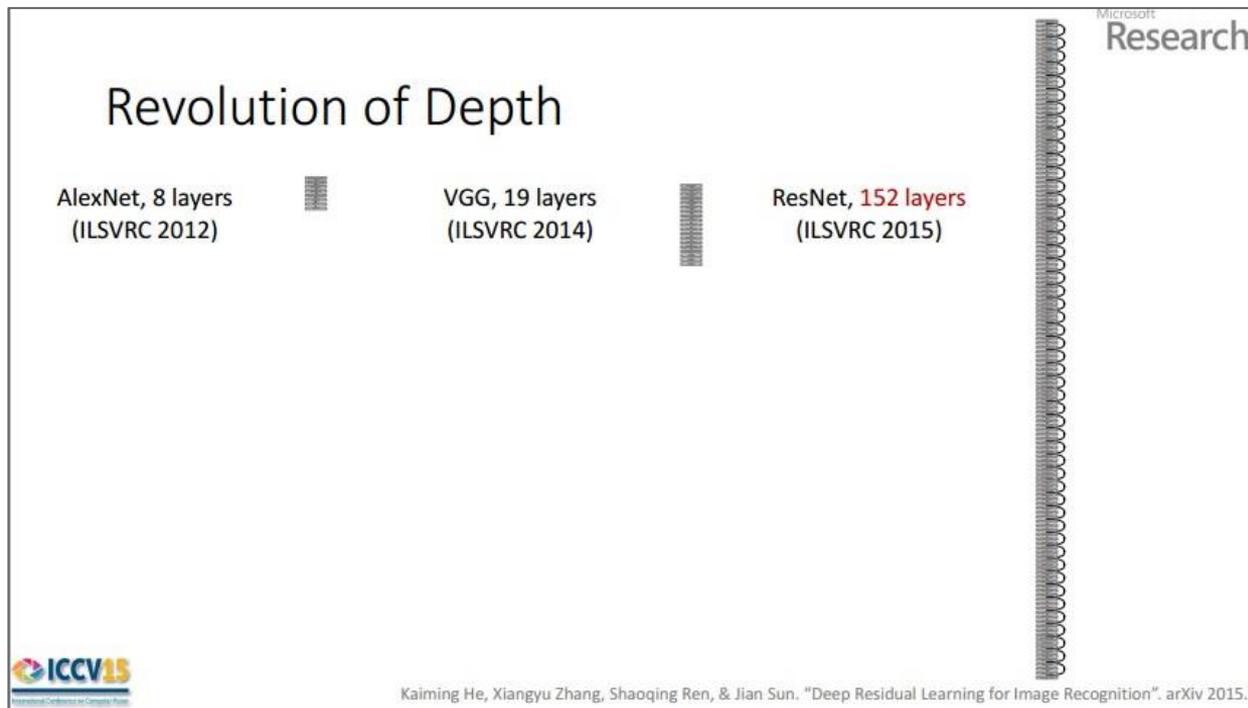


CIFAR-10 ResNets



Case Study: ResNet [He et al., 2015]

ILSVRC 2015 winner (3.6% top 5 error)



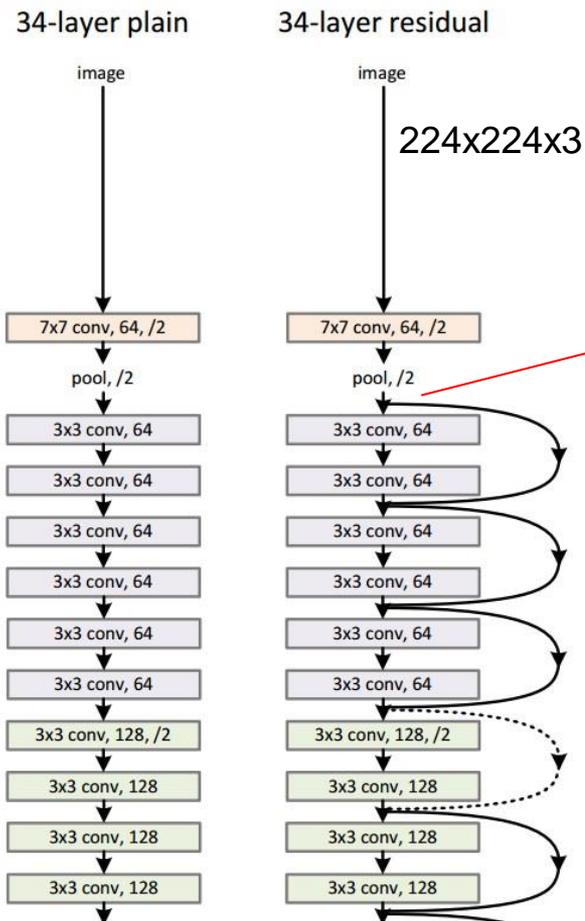
2-3 weeks of training on 8 GPU machine

at runtime: faster than a VGGNet! (even though it has 8x more layers)

(slide from Kaiming He's recent presentation)

Case Study: ResNet

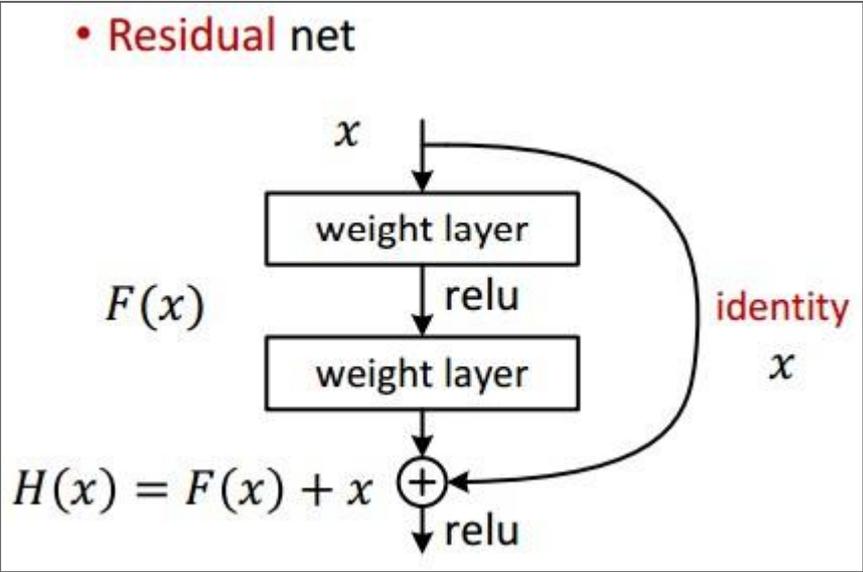
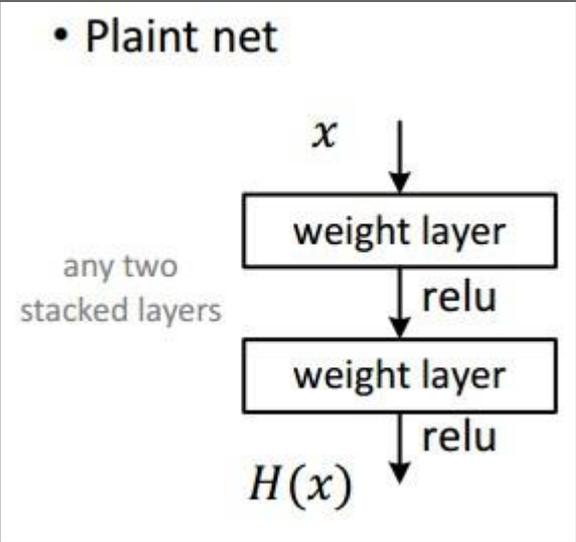
[He et al., 2015]



spatial dimension
only 56x56!

Case Study: ResNet

[He et al., 2015]



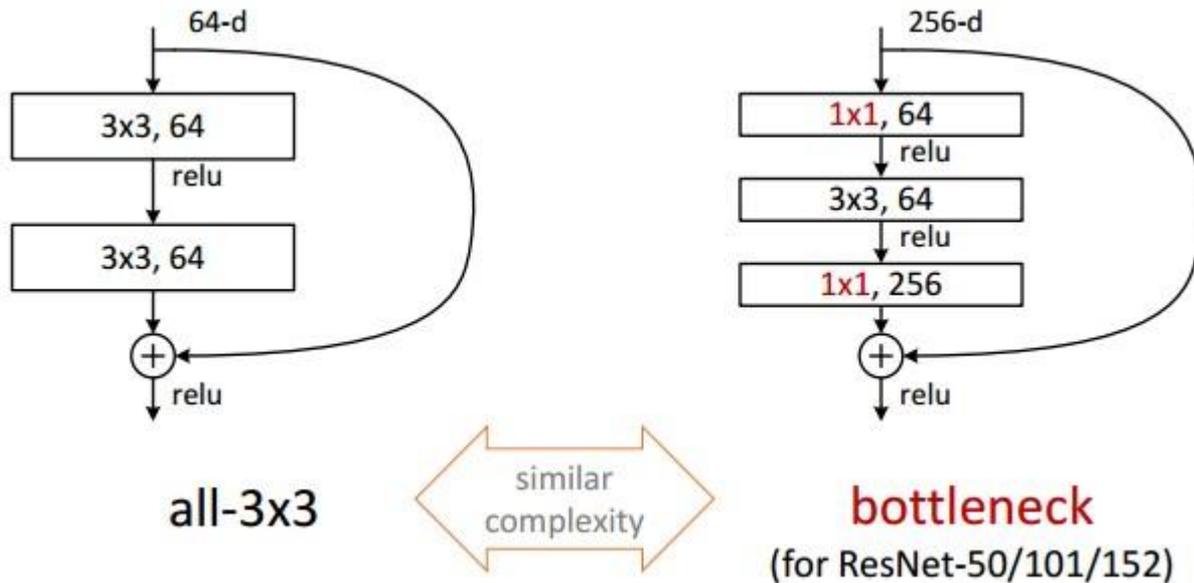
Case Study: ResNet

[He et al., 2015]

- Batch Normalization after every CONV layer
- Xavier/2 initialization from He et al.
- SGD + Momentum (0.9)
- Learning rate: 0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of $1e-5$
- No dropout used

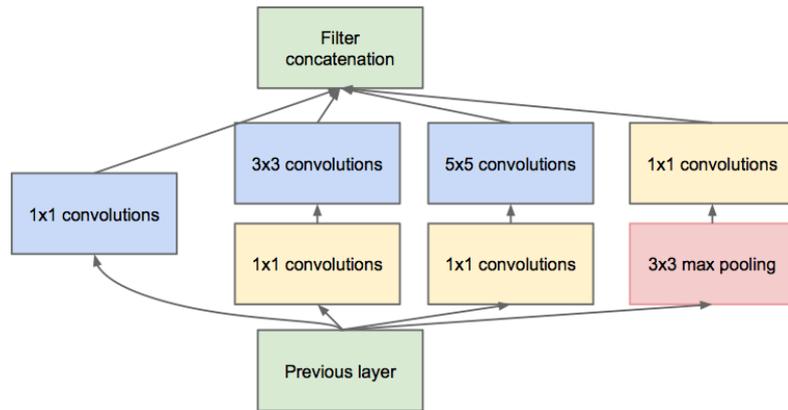
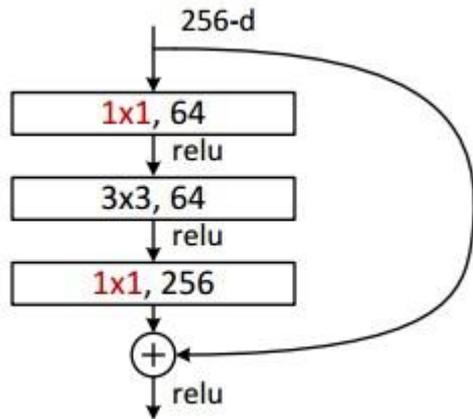
Case Study: ResNet

[He et al., 2015]



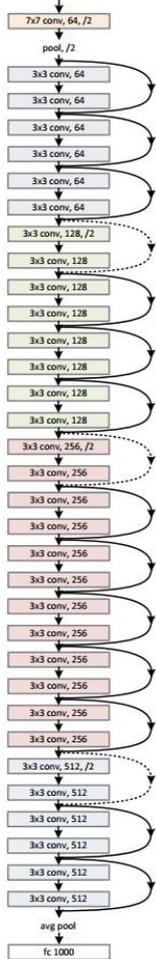
Case Study: ResNet

[He et al., 2015]



(this trick is also used in GoogLeNet)

Case Study: ResNet [He et al., 2015]



layer name	output size	18-layer	34-layer	50-layer	101-layer	152-layer
conv1	112×112	7×7, 64, stride 2				
conv2_x	56×56	3×3 max pool, stride 2				
		$\begin{bmatrix} 3 \times 3, 64 \\ 3 \times 3, 64 \end{bmatrix} \times 2$	$\begin{bmatrix} 3 \times 3, 64 \\ 3 \times 3, 64 \end{bmatrix} \times 3$	$\begin{bmatrix} 1 \times 1, 64 \\ 3 \times 3, 64 \\ 1 \times 1, 256 \end{bmatrix} \times 3$	$\begin{bmatrix} 1 \times 1, 64 \\ 3 \times 3, 64 \\ 1 \times 1, 256 \end{bmatrix} \times 3$	$\begin{bmatrix} 1 \times 1, 64 \\ 3 \times 3, 64 \\ 1 \times 1, 256 \end{bmatrix} \times 3$
conv3_x	28×28	$\begin{bmatrix} 3 \times 3, 128 \\ 3 \times 3, 128 \end{bmatrix} \times 2$	$\begin{bmatrix} 3 \times 3, 128 \\ 3 \times 3, 128 \end{bmatrix} \times 4$	$\begin{bmatrix} 1 \times 1, 128 \\ 3 \times 3, 128 \\ 1 \times 1, 512 \end{bmatrix} \times 4$	$\begin{bmatrix} 1 \times 1, 128 \\ 3 \times 3, 128 \\ 1 \times 1, 512 \end{bmatrix} \times 4$	$\begin{bmatrix} 1 \times 1, 128 \\ 3 \times 3, 128 \\ 1 \times 1, 512 \end{bmatrix} \times 8$
conv4_x	14×14	$\begin{bmatrix} 3 \times 3, 256 \\ 3 \times 3, 256 \end{bmatrix} \times 2$	$\begin{bmatrix} 3 \times 3, 256 \\ 3 \times 3, 256 \end{bmatrix} \times 6$	$\begin{bmatrix} 1 \times 1, 256 \\ 3 \times 3, 256 \\ 1 \times 1, 1024 \end{bmatrix} \times 6$	$\begin{bmatrix} 1 \times 1, 256 \\ 3 \times 3, 256 \\ 1 \times 1, 1024 \end{bmatrix} \times 23$	$\begin{bmatrix} 1 \times 1, 256 \\ 3 \times 3, 256 \\ 1 \times 1, 1024 \end{bmatrix} \times 36$
conv5_x	7×7	$\begin{bmatrix} 3 \times 3, 512 \\ 3 \times 3, 512 \end{bmatrix} \times 2$	$\begin{bmatrix} 3 \times 3, 512 \\ 3 \times 3, 512 \end{bmatrix} \times 3$	$\begin{bmatrix} 1 \times 1, 512 \\ 3 \times 3, 512 \\ 1 \times 1, 2048 \end{bmatrix} \times 3$	$\begin{bmatrix} 1 \times 1, 512 \\ 3 \times 3, 512 \\ 1 \times 1, 2048 \end{bmatrix} \times 3$	$\begin{bmatrix} 1 \times 1, 512 \\ 3 \times 3, 512 \\ 1 \times 1, 2048 \end{bmatrix} \times 3$
	1×1	average pool, 1000-d fc, softmax				
FLOPs		1.8×10^9	3.6×10^9	3.8×10^9	7.6×10^9	11.3×10^9

The input to the policy network is a $19 \times 19 \times 48$ image stack consisting of 48 feature planes. The first hidden layer zero pads the input into a 23×23 image, then convolves k filters of kernel size 5×5 with stride 1 with the input image and applies a rectifier nonlinearity. Each of the subsequent hidden layers 2 to 12 zero pads the respective previous hidden layer into a 21×21 image, then convolves k filters of kernel size 3×3 with stride 1, again followed by a rectifier nonlinearity. The final layer convolves 1 filter of kernel size 1×1 with stride 1, with a different bias for each position, and applies a softmax function. The match version of AlphaGo used $k = 192$ filters; [Fig. 2b](#) and [Extended Data Table 3](#) additionally show the results of training with $k = 128, 256$ and 384 filters.

policy network:

[19x19x48] Input

CONV1: 192 5x5 filters , stride 1, pad 2 => [19x19x192]

CONV2..12: 192 3x3 filters, stride 1, pad 1 => [19x19x192]

CONV: 1 1x1 filter, stride 1, pad 0 => [19x19] (*probability map of promising moves*)

Summary

- ConvNets stack CONV, POOL, FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Typical architectures look like
[(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K, SOFTMAX
where N is usually up to ~5, M is large, $0 \leq K \leq 2$.
 - but recent advances such as ResNet/GoogLeNet challenge this paradigm