EE106B/206B Lecture Notes

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Ch. 5.4 Grasp Planning

I. *Force Closure and Form Closure*

Form Closure = A sub-class of force closure that doesn't rely on friction ($\mu = 0$). [Proposition 5.3]

Form Closure implies Force Closure.

Assumptions about the object O:

- Rigid solid
- Exactly known geometry
- "Regular:" Constructive solid geometry (closed, compact) where O is the closure of its interior ("well-behaved" objects)
- $O \subset \mathbb{R}^3$

Let the object coordinate frame {O} origin be at center of mass.

Let $\Sigma = \partial(O)$ be the boundary of O – connected, piecewise smooth

Given a set of n contact points $C = \{c_i\}$ $i = 1 \dots n$ $c_i \in \Sigma$

Let $\Lambda(\Sigma)$ be the set of wrenches that can be applied to O with *frictionless* point contacts ($\boldsymbol{\mu = 0}$)

$\mathbf{p_{ci}}$ = location of c_i in {O}

$\mathbf{n_{ci}}$ = inward normal at c_i

$$\Lambda(\Sigma) = \left\{ \begin{bmatrix} n_{ci} \\ p_{ci} \times n_{ci} \end{bmatrix} \right\}$$

$\mathcal{F}(O, C)$ is true if C puts O into Form/Force Closure

If the convex hull of $\boldsymbol{\Lambda(\Sigma)}$ ***contains the origin {O}, then*** $\boldsymbol{\mathcal{F}(O, C)}$.

Also, let **p** be the *dimension of the wrench space,* $\mathbb{R}^p$.

- **p** = 3 in the plane
- **p** = 6 in space (3D)

Therefore, ***if*** $\boldsymbol{\Lambda(\Sigma)}$ ***positively spans*** $\mathbb{R}^{\boldsymbol{p}}$***, then*** $\boldsymbol{\mathcal{F}(O, C)}$.

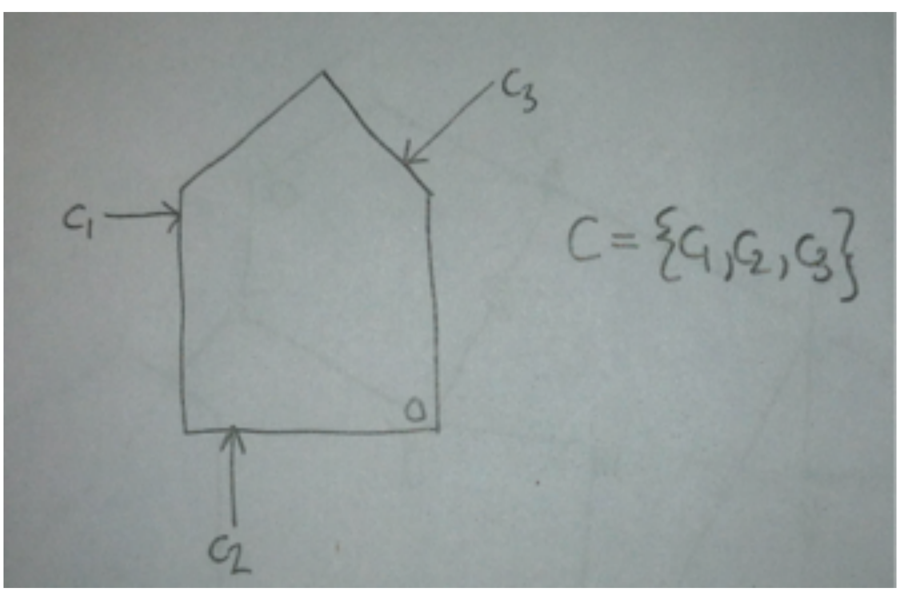
<u>**Exceptional Surface**</u>:

Object O with boundary ∂(O) such that it cannot be grasped (without friction).

Examples: sphere, circle, cylinder, etc.

$\neg\exists \subset \mathcal{F}(O, C)$

II. Grasp Planning in the Plane (μ = 0) – Geometric Intuition

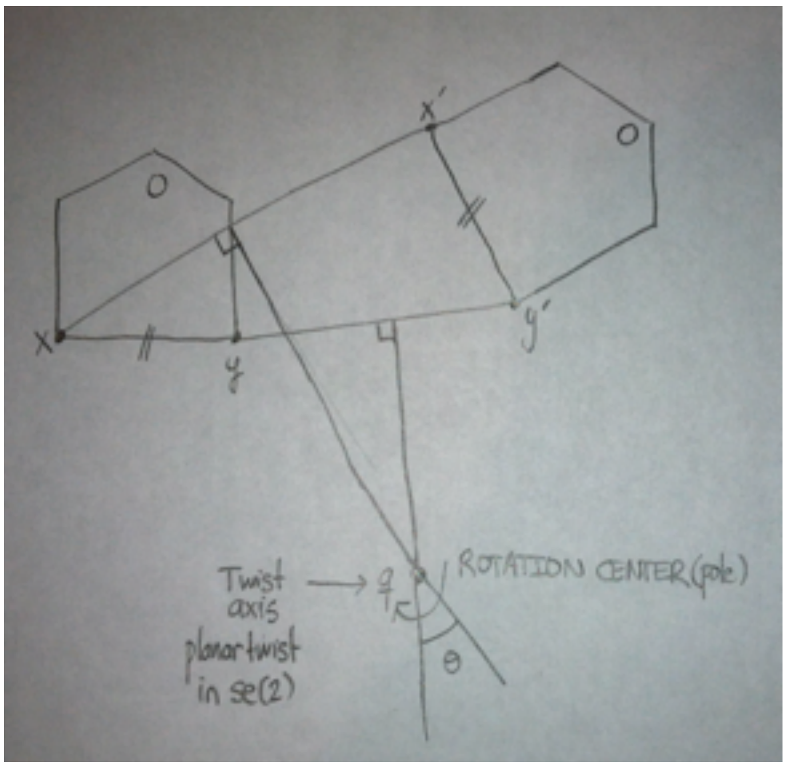


$\boldsymbol{\mathcal{F}(O, C)}$**?**

Analysis: Given O, C: is $\mathcal{F}(\boldsymbol{O}, \boldsymbol{C})$ true?

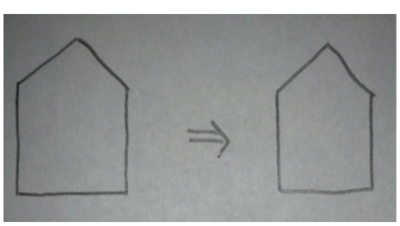
Synthesis: Given O, find C such that $\mathcal{F}(\boldsymbol{O}, \boldsymbol{C})$ is true.

I. Rigid Body Motion:



$$\xi = \begin{bmatrix} 2y \\ -2x \\ 1 \end{bmatrix} \text{ (pure rotation) or } \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \text{ (pure translation).}$$

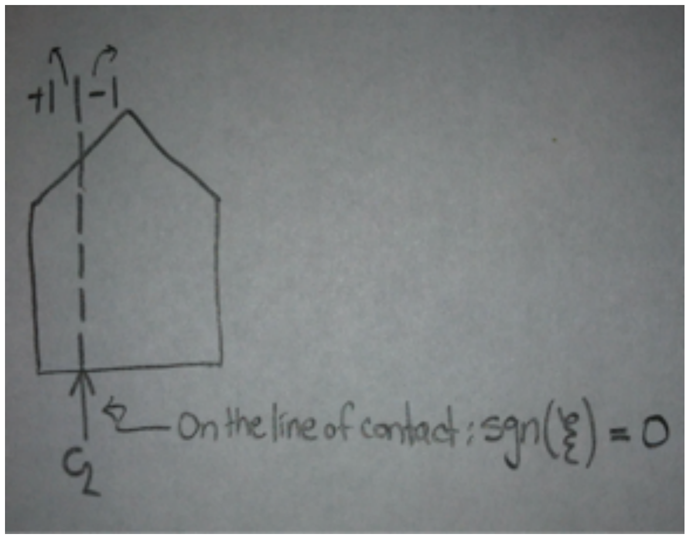
Pure translation:


Intersection at ∞.

II. Frictionless point contact: constraints on ξ

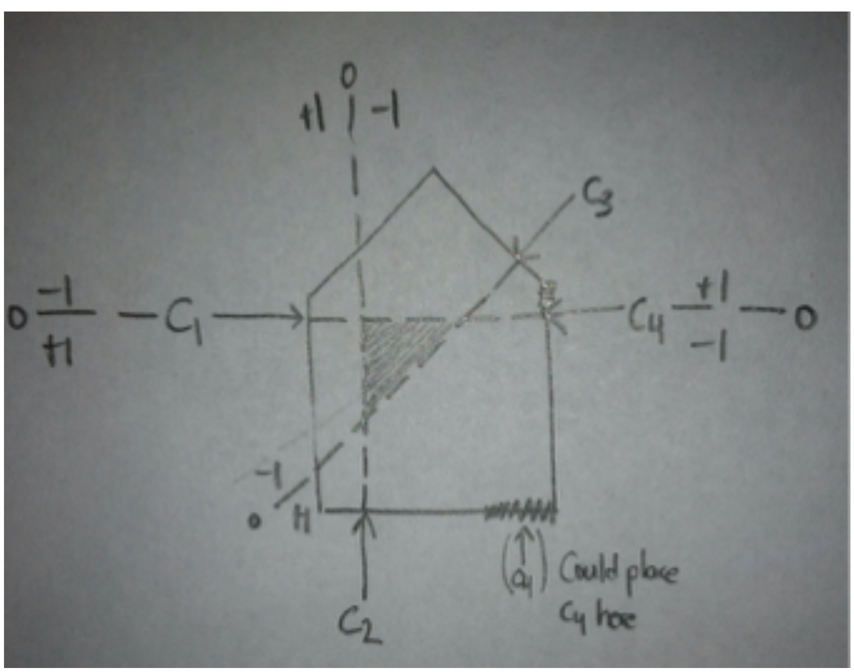
$$\text{Define } \operatorname{sgn}(\xi) = \begin{cases} +1 \; if \; \theta > 0 \\ 0 \; if \; pure \; translation \\ -1 \; if \; \theta < 0 \end{cases}$$

sgn(ξ) ≡ -1 in this halfplane



sgn(ξ) ≡ +1 in this halfplane

III. Rotation Center Locus: Multiple (frictionless) Contacts:



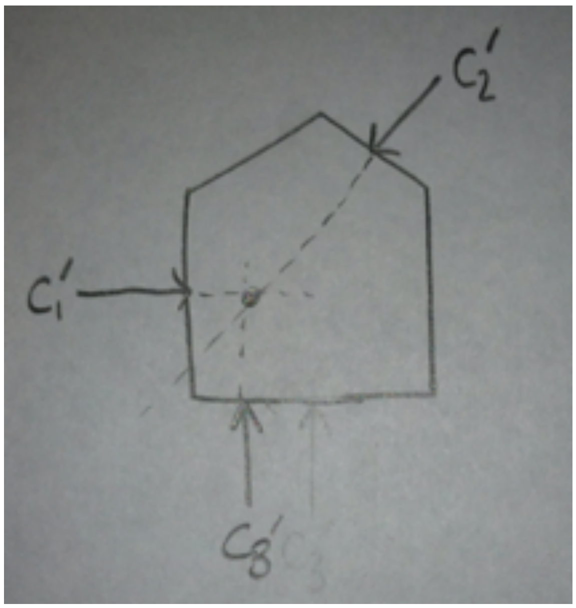
$C = \{c_1, c_2, c_3, c_4\}$

Let $\Xi = \{\xi_i\}$

If $\Xi = \varnothing$: $\boldsymbol{\mathcal{F}(O, C)}$

To eliminate the locus: place $\mathbf{p_4}$ anywhere on the jagged edges.

Are 3 contacts enough?



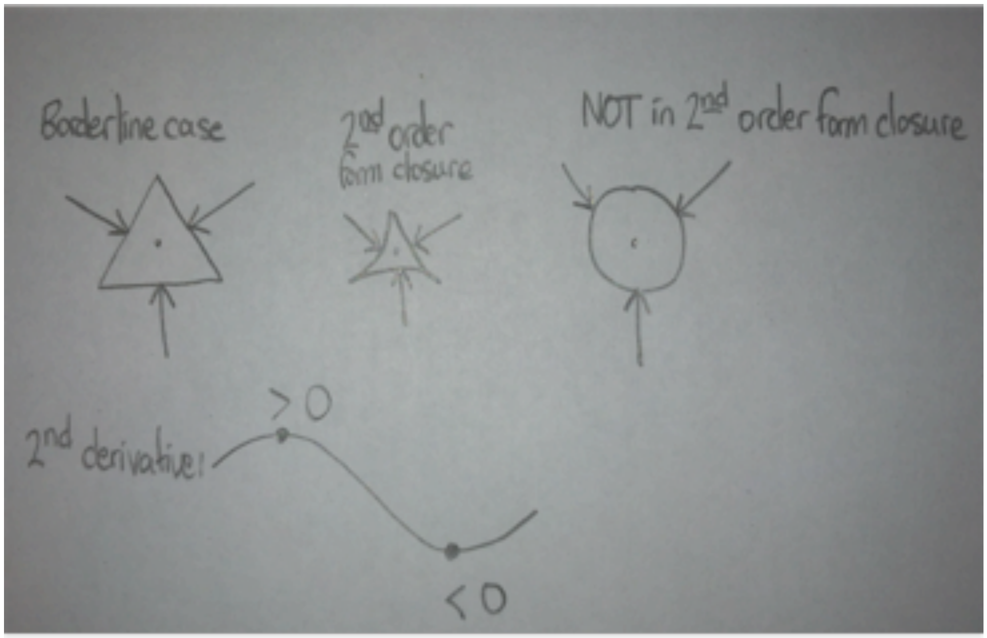
Is O in form closure? ***No***: The locus is a point with sgn(p) = ±1, so there is *infinitesimal* rotation.

∴ Need ≥ 4 contacts in the plane.

So far, we haven't been allowing contacts on the corners.

Contacts at concave vertices → very important, there is a lot of constraint there.

2nd Order Form Closure:

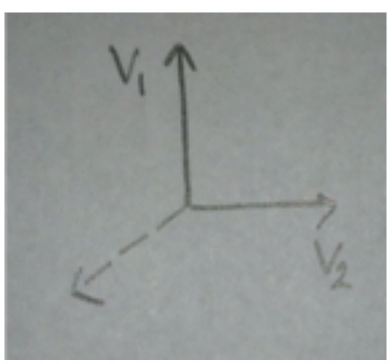


IV. **Number of Required Contacts**:

Given a set of vectors X = {v_1, …, v_k}, X positively spans $\mathbb{R}^p$ if and only if co(X) [Convex-hull of X] contains a neighborhood of the origin (pg. 255).

Theorem 5.4: (Caratheodory) 1911 (Greek)

At least p+1 vectors are necessary to positively span $\mathbb{R}^p$.



For p = 2: $\forall\, v_1, v_2 : -(v_1 + v_2)$ is outside the positive span of v_1, v_2

Theorem 5.5: (Steinitz – Jewish German)

Given a set of vectors that positively span $\mathbb{R}^p$, $\exists$ a subset of 2p or Fewer sufficient to positively span $\mathbb{R}^p$.

Recall: p = dimension of the wrench space

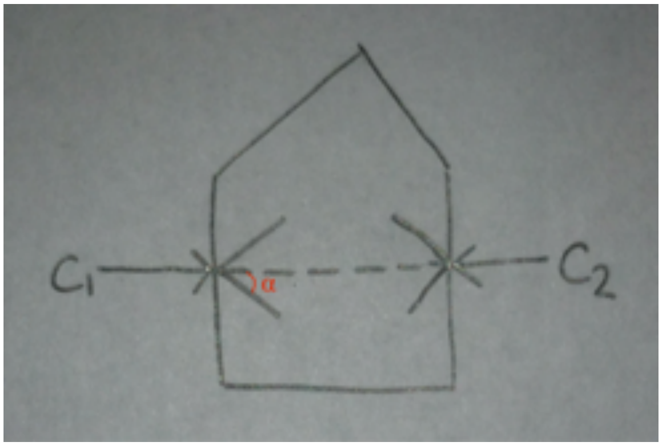
Table 5.3: Lower bounds on the number of fingers required to grasp an object.

Space	Object type	Lower	Upper	FPC	PCWF	SF
Planar	Exceptional	4	6	n/a	3	3
($p = 3$)	Non-exceptional			4	3	3
Spatial	Exceptional	7	12	n/a	4	4
($p = 6$)	Non-exceptional			12	4	4
	Polyhedral			7	4	4

V. **<u>Grasp planning in the plane ($\mu > 0$)</u>**

2 point contacts with friction (2PCWF) in plane.

Define <u>grasp axis</u>: $[p_1 - p_2]$



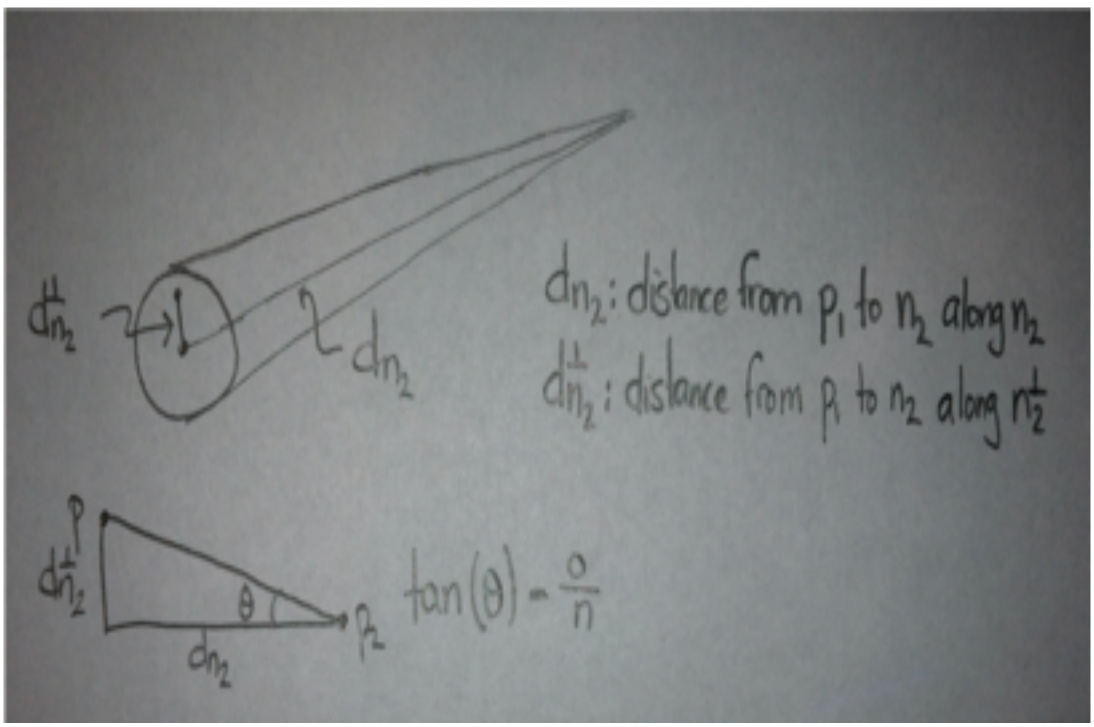
$\tan(\alpha) = \mu$

Theorem 5.6: PCWF

> G is in FC if and only if the grasp axis lies strictly inside both friction cones. [Nguyen '88]
>
> (Related to Def. 5.2, Prop. 5.1, 5.2, 5.3)

Extends to 2 point contacts with friction in 3D.

Theorem 5.7: Check both contacts individually. [Nguyen '88]



Check 1: Is c_1 inside Friction Cone 2?

Check 2: Contact 1 is stable if $\frac{d_{n_2}}{d^{\perp}_{n_2}} < \mu$.

If both are stable, $\boldsymbol{\mathcal{F}(O, C)}$.

i.e. Don't need to approximate the friction cone.

VI. Grasp Planning with Uncertainty (in Pose) in the Plane

Assume: Known object

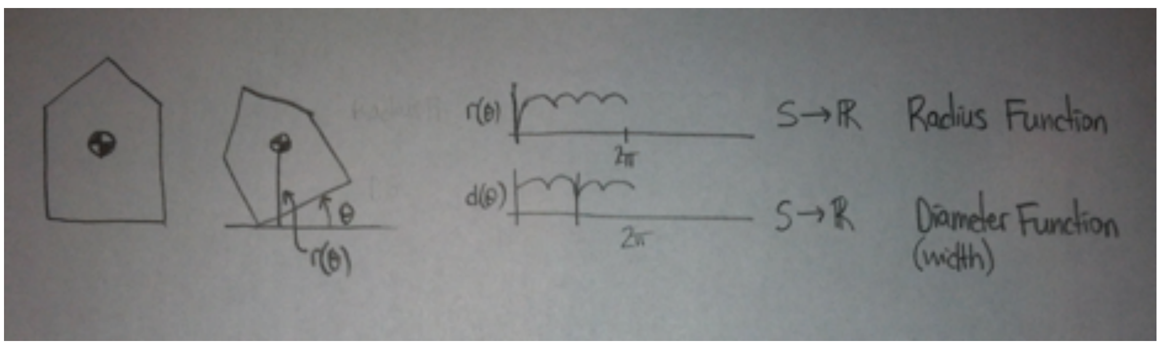
Pose: Not known precisely

Parallel-Jaw Gripper (pg. 11 Problem)

Orienting Polygonal Parts in the Plane: Algorithmica (1993)
Dr. Ken Goldberg

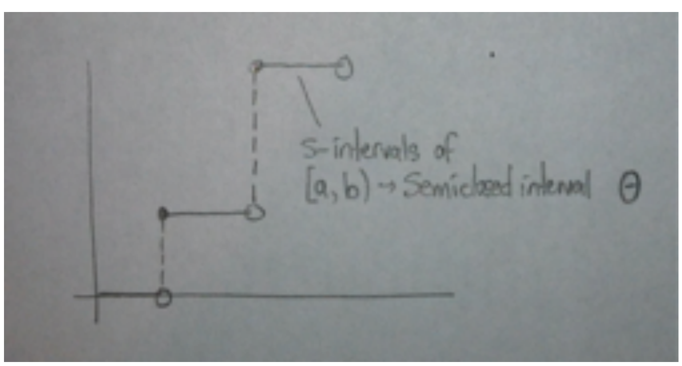
Convex hull of O

Radius Function and Diameter Function:



Squeeze function *s, s'* → *s'*

Piecewise constant monotone step function



Θ_x where θ_x is leftmost point

Symmetry in object → periodicity → aliasing