23 Feb 2016
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EE 106B/206B

NOTE TAKER

CH 5.4 GRASP PLANNING

I. FORCE CLOSURE : Proposition 5.3

"FORM CLOSURE" : ~~special~~ subclass of Force closure that doesn't rely on Friction. → if Form then Force C.

The ~~[illegible]~~ object : rigid solid with exact, known geometry

closed/compact, that is the closure of its interior.

connected, ~~piecewise smooth~~

$O \subset \mathbb{R}^3$ "regular" (CSG)

→ Let object coord frame be at com: $\{O\}$

Let $\Sigma = \partial O$ boundary of O : connected, piecewise smooth (no holes?)

Given set of n contacts $C = \{c_i\}$: $c_i \in \Sigma$ → Let $\Lambda(\Sigma)$ set of all wrenches that can be applied to O w/ frictionless point contacts

for n contacts: $C, \{c_1, \ldots c_i \ldots c_n\} \in \Sigma$: p_{ci} location of contact i in frame $\{O\}$; n_{ci} inward pointing normal at contact i in frame $\{O\}$

$\mu = 0$: $$\Lambda(\Sigma) = \left\{ \begin{bmatrix} n_{ci} \\ p_{ci} \times n_{ci} \end{bmatrix} \right\}$$

with $\mu = 0$: For force closure (and form closure): $\mathcal{F}(O,C)$, if: "O to be grasped (w/o friction)" : convex hull of $\Lambda(\Sigma)$ must contain ~~the origin~~ origin of $\{O\}$ $\equiv$ positively span $\mathbb{R}^p$ (see p. 225)

p = dim of wrench space

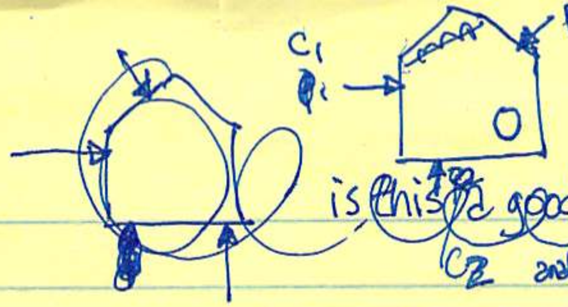
Exceptional Surface : Σ such that $\neg \exists C$

cannot be grasped (w/o friction) eg. sphere, circle, cylinder, ...

O: $\neg \exists C \; \mathcal{F}(O,C)$

~~5.4.7~~ CONSTRUCTING FORCE CLOSURE GRASPS

$\mu=0$: FORM CLOSURE



$\mu=0$:

$\mathcal{F}(O,C)$?

is this a good grasp

1. Given O, C, is $\mathcal{F}(O,C)$ true? — analysis
2. Given O, Find C such that $\mathcal{F}(O,C)$. — synthesis

II ~~Frictionless~~ Grasp Planning in the Plane ($\mu=0$)

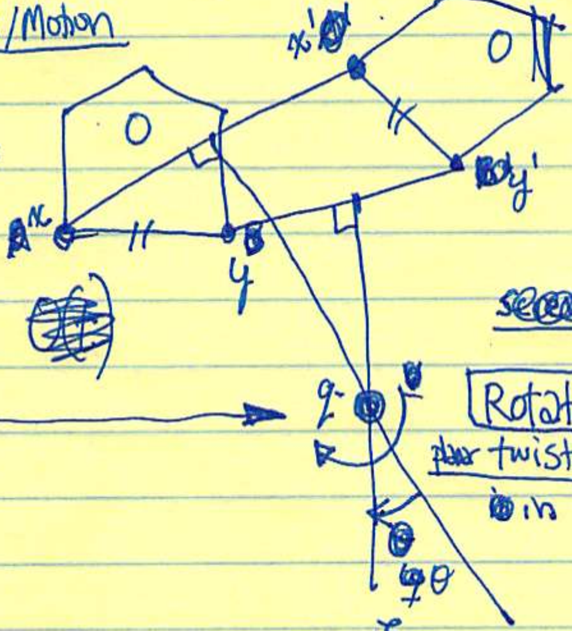
(geometric intuition)

1. Rigid ~~Planar Motion~~ Finite Body ~~Displacement~~ /Motion

Ch2:

Chasles' Th:

MASON



twist axis:

Axis of twist $\xi = (v,\omega)$

Rotation Center (pole)

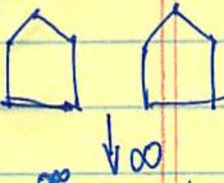
planar twist in $se(2)$

$\xi = \begin{bmatrix} \cdot \end{bmatrix} \begin{bmatrix} q_y \\ -q_x \\ 1 \end{bmatrix}$ or $\begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix}$

pure rotation p.76

pure translation

consider:



$\downarrow \infty$

Pure Translation: ξ at infinity: (p.46)

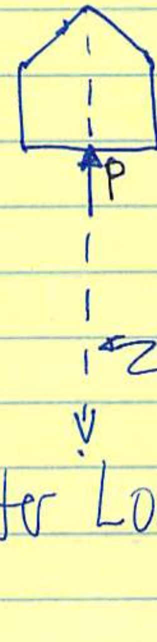
2. one Frictionless point contact: constraints on ξ:

define: $\mathrm{sgn}(\xi) := \begin{cases} +1 & \text{if } \theta>0 \\ 0 & \text{if pole at } \infty \\ -1 & \text{if } \theta<0 \end{cases}$

$+1 \quad -1 = \mathrm{sgn}(\omega)$

$= \mathrm{sgn}(\xi) :\equiv \mathrm{sgn}(\omega)$

in this halfplane



in this halfplane, constraint is that:

$\mathrm{sgn}(\omega) \equiv +1$

~~$\mathrm{sgn}(\theta) < 0$~~

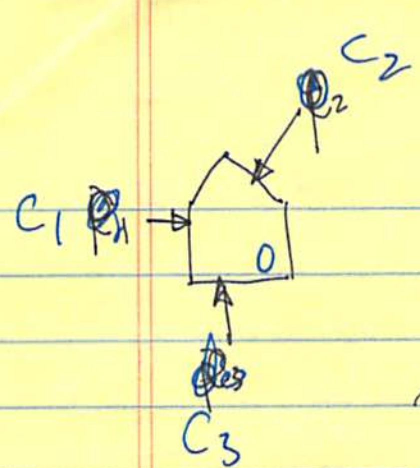
$\mathrm{sgn}(\omega) \equiv -1$

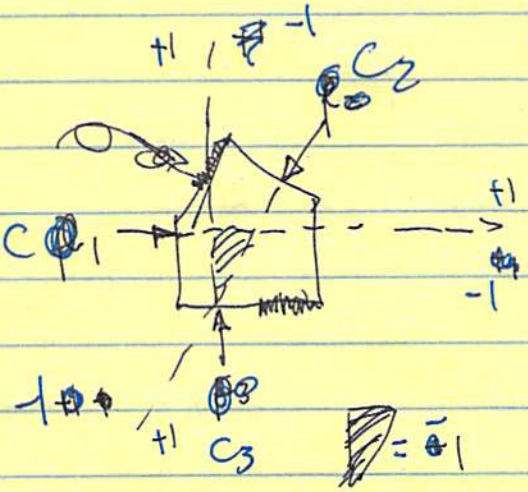
ξ on the Line of contact: $\mathrm{sgn}(\xi) = 0$

3. Rotation Center Locus: multiple Contacts

~~GRASP PLANNING w/~~

FRICTIONLESS MULTIPLE CONTACTS





Let $\Xi = \{\xi_i\}$

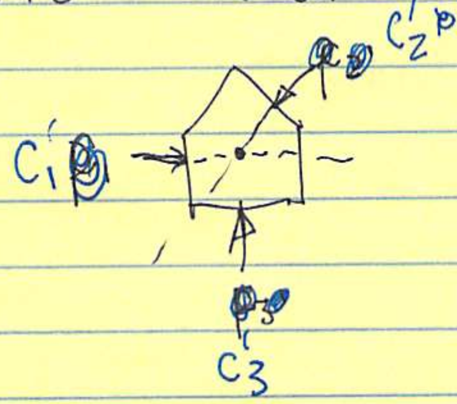
Thm: if $\Xi = \emptyset$: $FC(O,C)$

TO ELIMINATE THIS LOCUS:

PLACE R_4 ANYWHERE ON LOWER RIGHT EDGE.

HW: GIVEN $O = \langle(3,2),(-4,2),(-1,0),(-1,-5),(0,-5)\rangle$ construct a FC GRASP USING ROTATION CENTERS

ARE 3 ENOUGH?



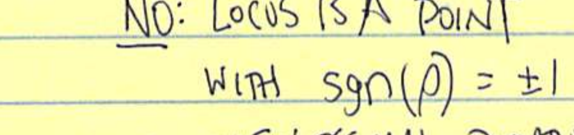
$\mathcal{F}^0(O,C)$?

IS O in FORM CLOSURE?

NO: LOCUS IS A POINT WITH $\mathrm{sgn}(\rho) = \pm 1$ SO INFINITESIMAL ROTATION.

$\therefore$ NEED ≥ 4 CONTACTS IN PLANE.

(BTW, YES: 2^{nd} order form closure)



local curvature of contacts

Given A set of vectors $X=\{v_1,\dots v_K\}$

Prop. 5.3

They positively span $\mathbb{R}^p$ iff $co(X)$ Convex-hull of X, contains a neighborhood of the origin.

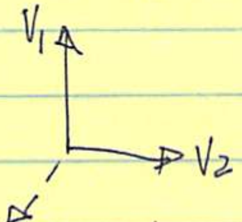
(p. 225)

~~(Recall p : dimension of wrench space)~~

Theorem 5.4: (Caratheodory) 1911 (Greek) (?)

at least $p+1$ vectors are necessary to ~~needed to~~ positively span $\mathbb{R}^p$

for $p=2$,



$\forall v_1, v_2: -(v_1+v_2)$ is always outside the pos. span of v_1, v_2

Theorem 5.5: (Steinitz) Jewish German

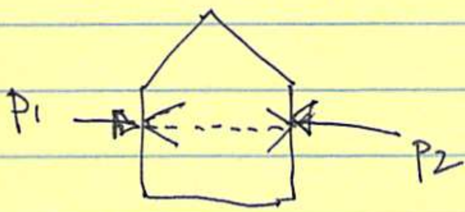
Given a set of vectors that positively span $\mathbb{R}^p$, $\exists$ a subset of $2p$ or fewer sufficient to positively span $\mathbb{R}^p$

RECALL p = dimension of wrench space

Table 5.3:

5.4.2 GRASP PLANNING IN THE PLANE ($\mu > 0$)

2 POINT CONTACTS W/FRICTION (2PCWF) in Plane



define <u>grasp axis</u>: $[p_1 - p_2]$

<u>TH 5.6</u> PCWF [83] Nguyen '88

related to Def. 5.2 Prop. 5.1, 5.2, 5.3

G is in FC <u>iff</u> grasp axis lies strictly inside both friction cones

see p. 232-3 Fig 5.13

Extends to

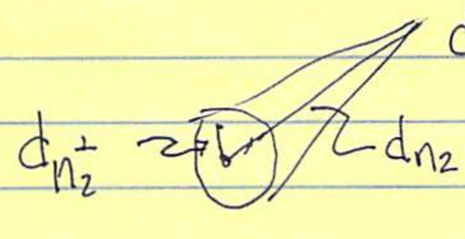
2 point contacts w/friction in 3D

THM 5.7: [83] Nguyen '88

CHECK BOTH CONTACTS INDIVIDUALLY:

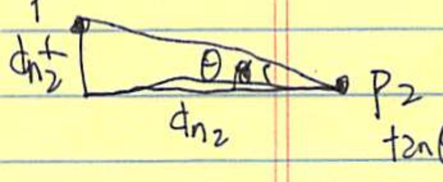
CHECK 1: p_1 : INSIDE FC_2? easy to check:

construct n_2



d_{n_2}: distance from p_1 to n_2 along n_2

$d_{n_2^\perp}$: distance from p_1 to n_2 along $n_2^\perp$



Contact at p_1 is stable if

$d_{n_2} / d_{n_2^\perp} < \tan \mu$

ie. don't need to approx friction cone.

CHECK 2 : P_2 inside FC_1 ?

if both True, then 3D FORCE CLOSURE

GRASP PLANNING W/UNCERTAINTY (IN POSE) IN PLANE

assume : KNOWN OBJECT

POSE : NOT KNOWN PRECISELY

p.11 Problem Parallel-Jaw Gripper

Orienting Polygonal Parts in the Plane : Algorithmica (1993)

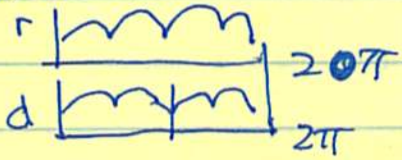
K. GOLDBERG

$\boxed{\text{Convex hull of O}}$

radius function

diameter function

(width)

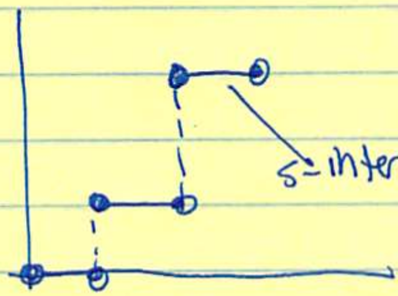


$S^1 \to \mathbb{R}$

$S^1 \to \mathbb{R}^+$

p 9. Squeeze function s, $S^1 \to S^1$

piecewise const monotone step function



Θ_x where θ_x is leftmost point

Symmetry in object → periodicity → aliasing