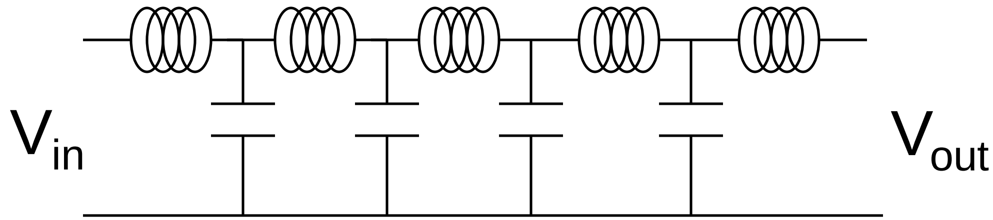
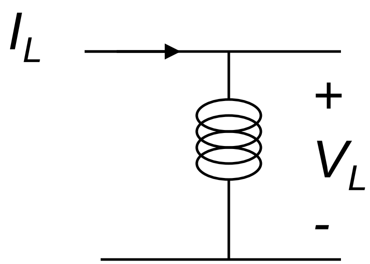
Active Filter Implementation

- **Active Filter Implementation**
  - Filter implementation
  - Gm-C
  - Cascade Opamp-RC
  - Ladder
    - » Direct replacement
    - » Signal-flow graph (SFG)
- **References**

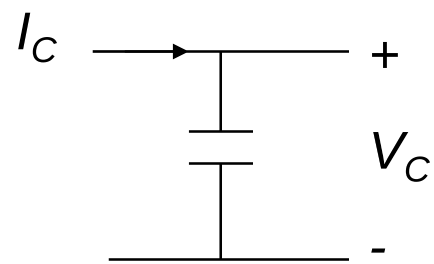
What do you need to implement a filter?



Passive low-pass filter



$$\boxed{I_L = \frac{1}{L}\int V_L dt}$$

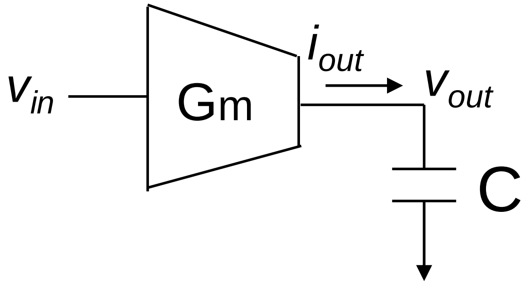


$$\boxed{V_C = \frac{1}{C}\int I_C dt}$$

- To build filters, you need integrators.

Active integrators:

Gm-C topology

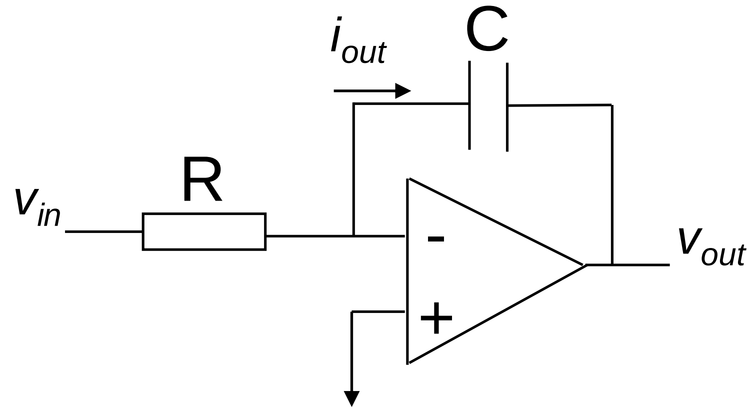


$$I_{out} = G_m V_{in}$$

$$V_{out} = \frac{1}{C}\int I_{out} dt = \frac{I_{out}}{sC}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{G_m}{C}\frac{1}{s}$$

Opamp-RC topology



$$I_{out} = \frac{V_{in}}{R}$$

$$V_{out} = -\frac{1}{C}\int I_{out} dt = \frac{I_{out}}{sC}$$

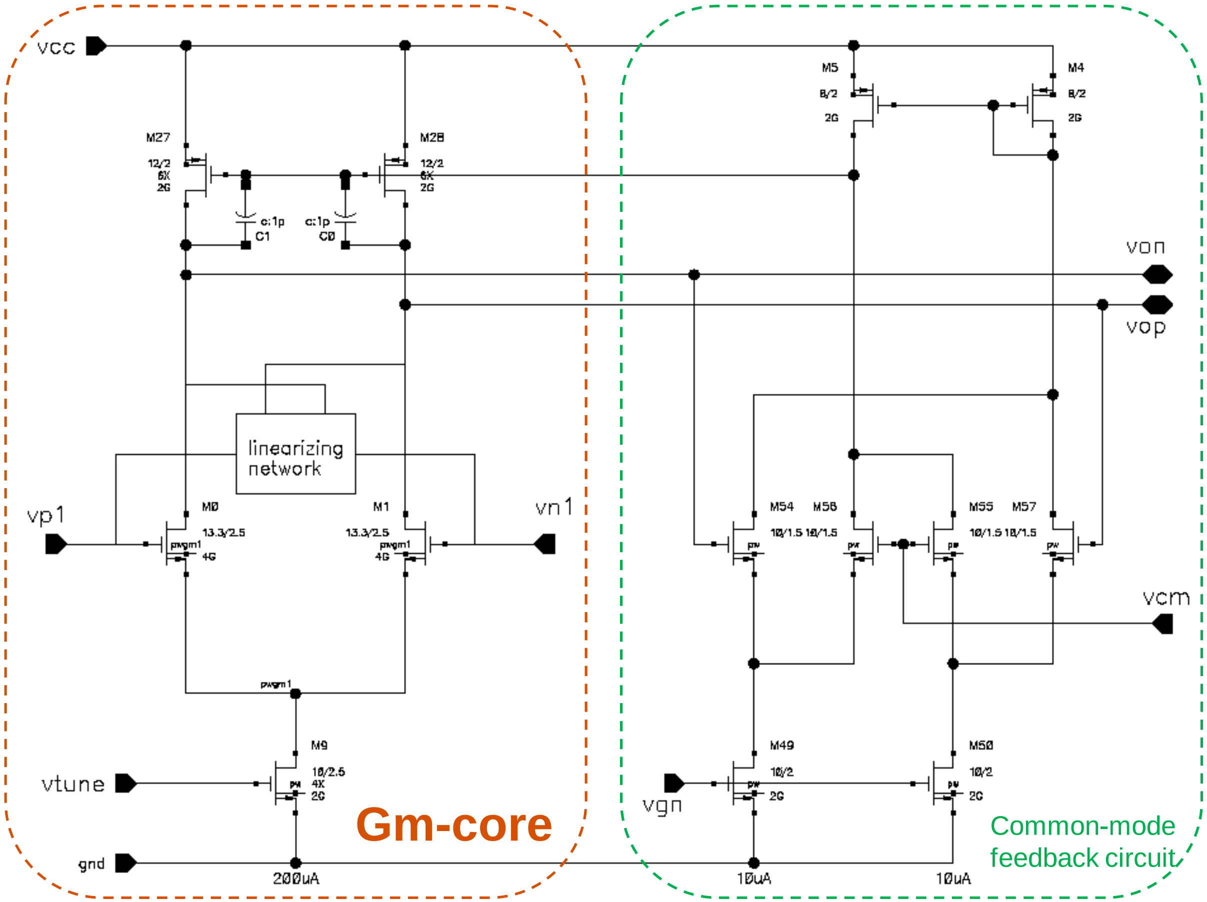
$$\Rightarrow \frac{V_{out}}{V_{in}} = -\frac{1}{RC}\frac{1}{s}$$

Gm-C implementation:

Gm-C implementation is as good as the Gm cell!!!

- fast! Gm-C implementation in general is an open-loop system, so its bandwidth is high.
- noise is set by the Gm cell input referred noise. Gm cell noise does not necessarily increase linearly with decreased Gm.
- linearity is limited by how linear the Gm cell is. Fast Gm cells do not use closed loop linearizing techniques, rather rely on simple techniques such as degeneration and harmonic cancellation. Therefore, fast Gm cells have limited linearity
- supply current can be made low if linearity requirement is not too stringent.
- area is mainly capacitor limited for low-frequency filters (<2MHz). However, for higher bandwidth filters (WLAN for e.g.), area is both capacitor and Gm cell limited, but can be smaller than opamp-RC.
- Performance variation over PVT is relatively large without careful design and proper biasing (requiring wider tuning range) compared to opamp-RC

Example of a Gm-C cell:



Linearizing network can be as simple as degeneration resistors or as complex as harmonic cancellation scheme, or even an opamp-based feedback network! It is a tradeoff between Gm-cell linearity and speed

opamp-RC implementation:

Opamp-RC implementation is as good as the opamp!!!

● relatively slow! Opamp-RC implementation is a closed-loop system, so its bandwidth is limited (by the opamp GBW).

● noise is set by kT/C, so lower noise means higher cap area (assuming opamp excess noise is small). This can be shown as follows:

resistor noise is: $4kTR\Delta f$

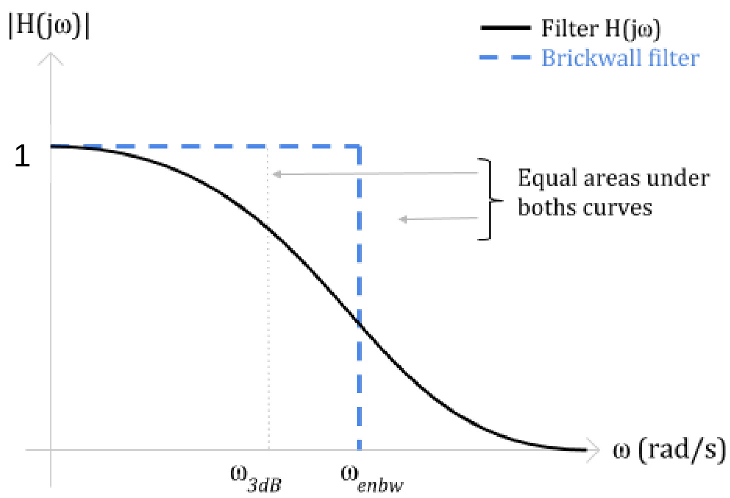
where Δf is the equivalent noise bandwidth (not the filter 3dB bandwidth)

the single-pole 3dB corner is: $\omega_c = \dfrac{1}{RC}$ rad/s $= \dfrac{1}{2\pi RC}$ Hz

the noise bandwidth for a single RC pole, ω_{enbw} is: $\displaystyle\int_0^\infty \left(\frac{1}{1+\frac{\omega}{\omega_c}}\right)^2 d\omega$

$= \dfrac{\pi}{2RC}(rad/s)$ or $\dfrac{1}{4RC}$ Hz. This is ~1.57x the filter 3dB BW

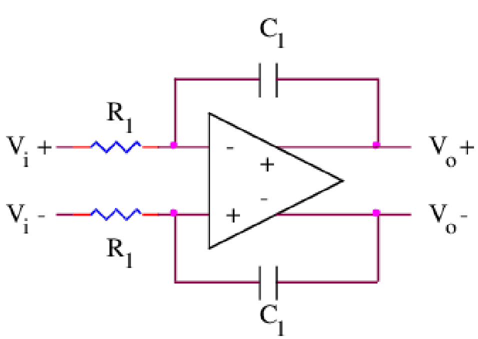
therefore, the integrated noise is: $4kTR\dfrac{1}{4RC} = \dfrac{kT}{C}$



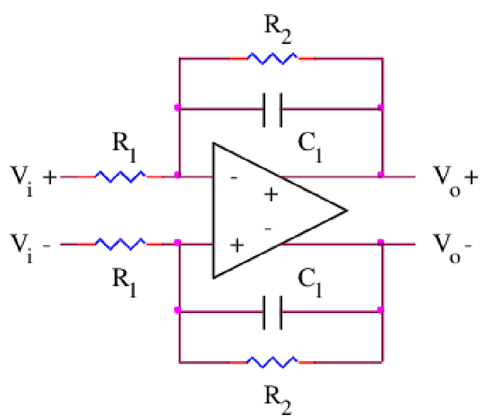
you can repeat the exercise for second order or higher. The conclusion is to reduce filter noise you need to increase C

Generally, opamp excess noise is 2~7nv/rt(Hz).

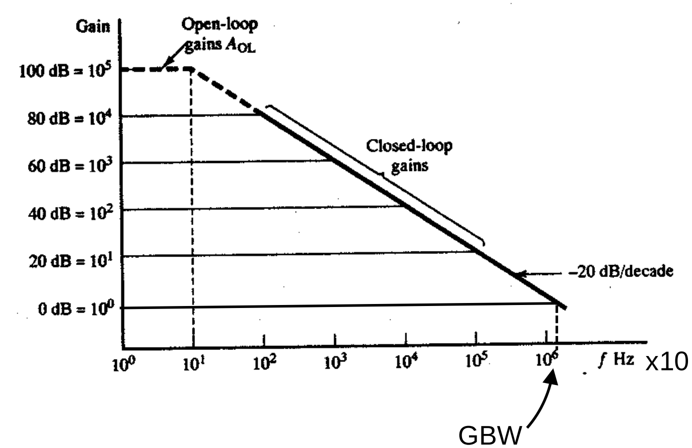
- linearity is limited by opamp loop gain at the blocker frequency. Therefore, for low-frequency filters, linearity of opamp-RC is very high.
- supply current is set by getting opamp gain-BW product to be ~100 times the filter bandwidth with >60dBc THD for 0.5Vpp signal.
- If the opamp has a Gain-BW product of X times the filter bandwidth, then the opamp has an open loop gain of 10log(X) at the filter 3dB frequency (assuming dominant-pole compensation for opamp)
- Die area is dictated by the required noise, which is mainly capacitor limited. This one a major drawback, especially for very low frequency filters because you need two floating capacitors per opamp/pole!



Loss-less integrator

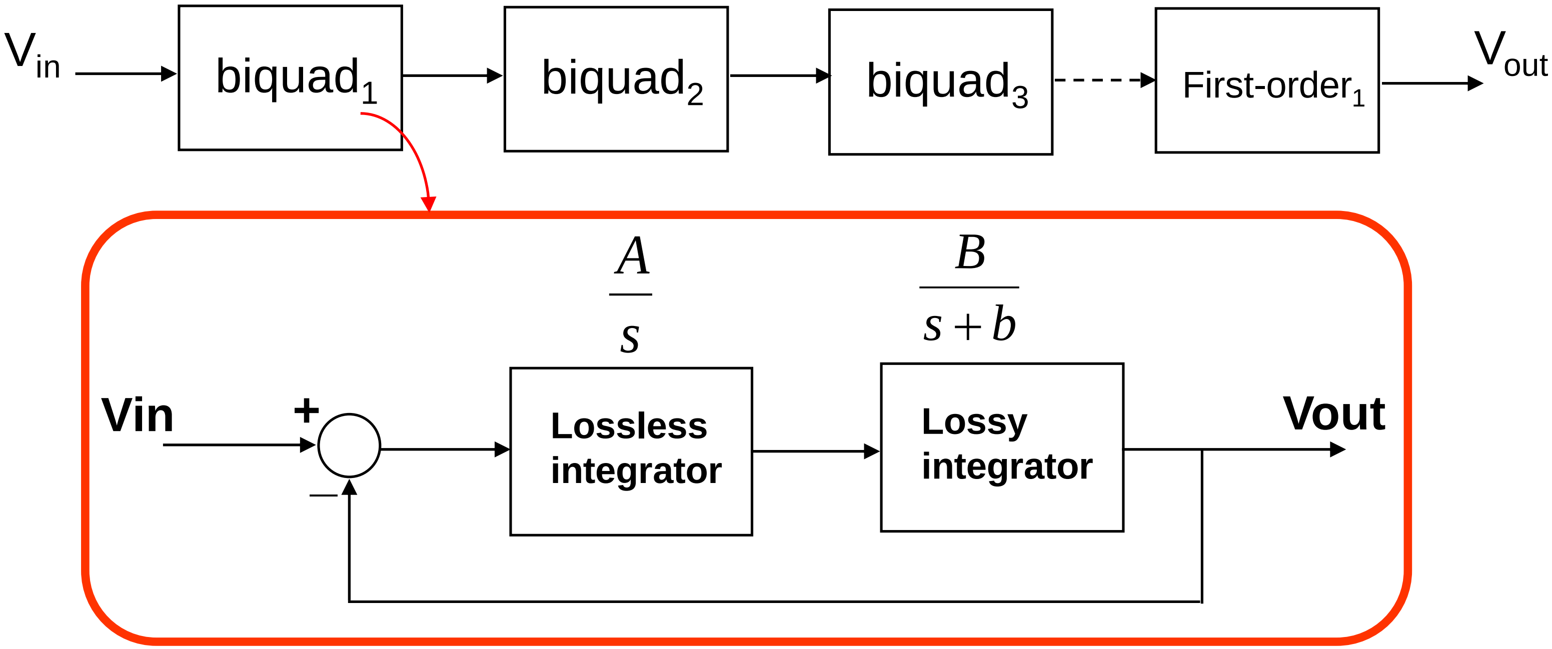


Lossy integrator



Filter implementation using cascade opamp-RC

- The basic building block of a cascade opamp-RC filter is <u>the biquad</u>



$$\frac{V_{out}}{V_{in}} = H(s) = \frac{T}{1+T} = \frac{\frac{A}{s}\frac{B}{s+b}}{1+\frac{A}{s}\frac{B}{s+b}} = \frac{AB}{s^2+bs+AB}$$

a lossy integrator is also called “first order” function.

Opamp-RC biquad:

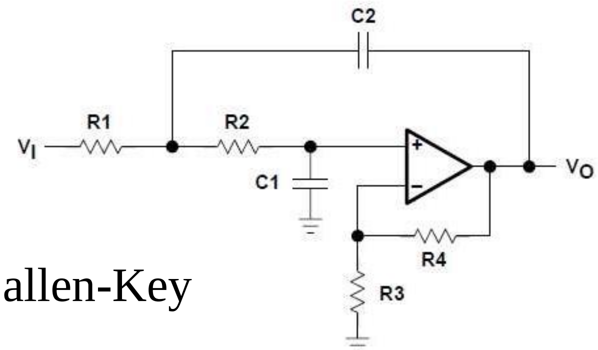
There are several opamp-RC biquads invented by designers, which carried their names. Some examples are:

- Sallen-Key biquad (one opamp)
- Thomas Tow I and II biquads (3 opamps, can be 2 for differential design)
- Ackerberg-Mossberg biquad (3 opamps, can be 2 for differential design)
- Delyiannis-Friend biquad (one opamp)
- multi-feedback biquad (one opamp)

These biquads differ from each other in:

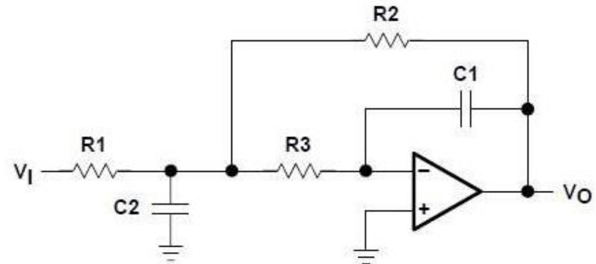
- The number of opamps needed (some have only one opamp, others two or three). This is a big deal for discreet design
- the total size of capacitance needed to realize the biquad
- realizable Q vs capacitance area
- swing at internal nodes

Examples of opamp-RC biquads:



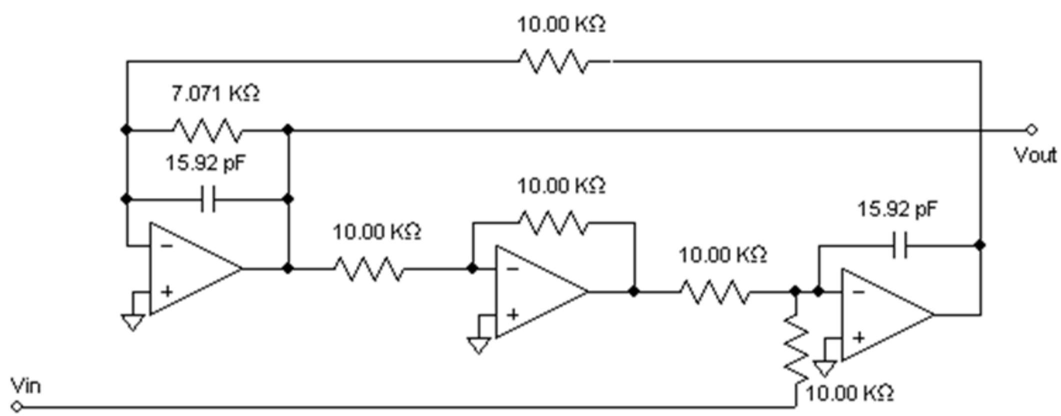
Sallen-Key

$$H(f) = \frac{\frac{R3+R4}{R3}}{(j2\pi f)^2(R1R2C1C2) + j2\pi f\left(R1C1 + R2C1 + R1C2\left(-\frac{R4}{R3}\right)\right) + 1}$$



$$H(f) = \frac{\frac{-R2}{R1}}{(j2\pi f)^2(R2R3C1C2) + j2\pi f\left(R3C1 + R2C1 + \left(\frac{R2R3C1}{R1}\right)\right) + 1}$$

Multi-feedback



Thomas-Tow II

Thomas-Tow I → move Vin of Thomas-Tow II to lossy integrator

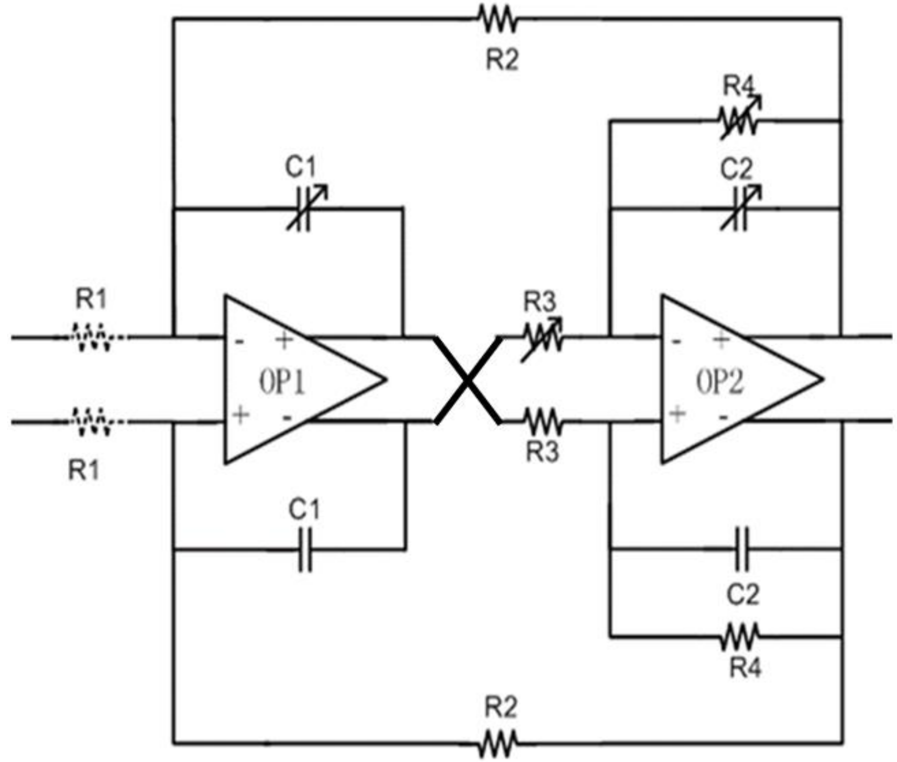
Examples of opamp-RC biquads, Cont:

$$H(s)_{LPF\_biquad} = \frac{A_V}{\dfrac{s^2}{\omega_0^2} + \left(\dfrac{1}{\omega_0 Q}\right)s + 1}$$

$$A_V = \frac{R_2}{R_1}$$

$$\omega_0 = \sqrt{\frac{1}{R_2 R_3 C_1 C_2}}$$

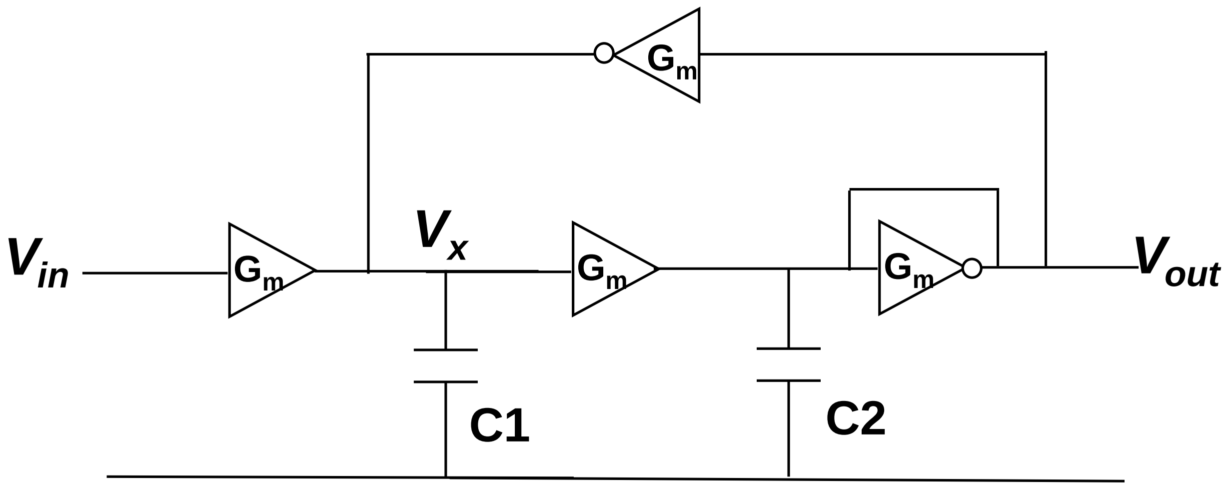
$$Q = R_4 C_2 \sqrt{\frac{1}{R_2 R_3 C_1 C_2}}$$



Resistor R1 is usually used to change filter gain without impacting its transfer function. C1 and C2 are used to tune the filter bandwidth without impacting its gain.

You can also chose to tune resistors (R2 and R3) to change the filter bandwidth while keeping C1 and C2 fixed. In this case you need to also tune R1 and R4 to keep filter gain and Q fixed

GmC biquad example:



Example of Gm-C biquad implementation. Assuming equal transconductors, one can find:

$$V_x = \frac{g_m}{sC_1}\left(V_{in} - V_{out}\right),$$

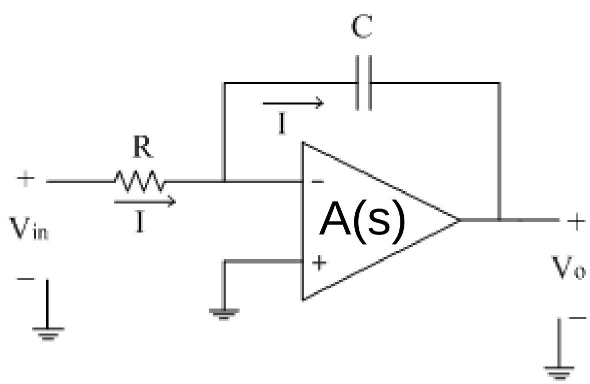
$$V_{out} = \frac{g_m}{sC_1 + g_m} V_x$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{g_m^2}{C_1C_2s^2 + g_mC_1s + g_m^2}$$

Realizable biquad Q with active integrators:

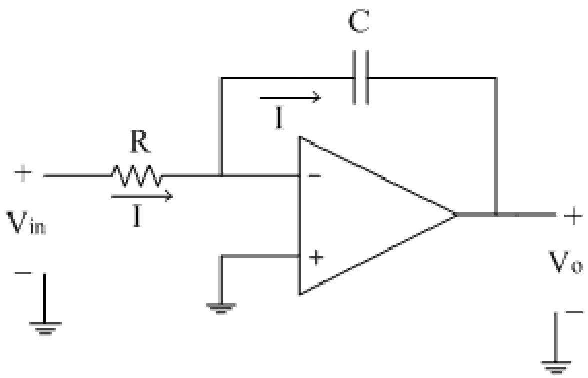
$$H(s)_{LPF\_biquad} = \frac{\omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

- The sharper the filter rejection is, the higher its Q becomes. This reduces the damping coefficient of the “*s*” term in the biquad equation above.
- The maximum biquad Q that can be realized with active integrators usually is <5 due to the lossless integrator excess phase due to opamp finite GBW (finite Gm output impedance and finite BW for Gm-C realization).



Opamp GBW impacts biquad Q

Realizable biquad Q with active integrators:



Ideal opamp

$$\frac{V_0}{V_{in}} = -\frac{1}{RCs}$$

real opamp has a finite gain BW, which can be written as:

$A(s) = \frac{\omega_u}{s+P_0}$; where ω_u is the opamp unity gain BW, P_0 is the opamp pole due to internal compensation

$$\Rightarrow \frac{V_0}{V_{in}} = -\frac{1}{RCs + (1+RCs)/A(s)} = -\frac{1}{RCs + \frac{s+P_0}{\omega_u}(1+RCs)}$$

Lossy integrator

- Real opamp has excess phase (due to its finite GBW and none-dominant pole) that causes the integrator to become lossy, limiting the max realizable Q of a biquad, especially at higher frequencies

Biquad-based cascaded filter realization flow:

1. find the s-domain representation of the filter factored into second-order terms.

$$H(s)=\frac{2.72\times10^{35}}{s^5+2.23\times10^{7}s^4+5.45\times10^{14}s^3+6.6\times10^{21}s^2+6.07\times10^{28}s+2.72\times10^{35}}$$

$$=\left(\frac{1.42\times10^{14}}{s^2+1.1\times10^{7}s+1.42\times10^{14}}\right)\left(\frac{2.5\times10^{14}}{s^2+3.58\times10^{6}s+2.5\times10^{14}}\right)\left(\frac{7.65\times10^{6}}{s+7.65\times10^{6}}\right)$$

2. chose a biquad topology (Opamp-RC based or Gm-C, depending on filter specs such as linearity, bandwidth ..etc.).

3. for the chosen topology, choose the biquad implementation (two lossy Gm-C integrators for Gm-C implementation for example, or Thomas-Tow II for opamp-RC implementation ..etc.). This is set by the biquad Q, noise peaking limit, max number of opamps ..etc.

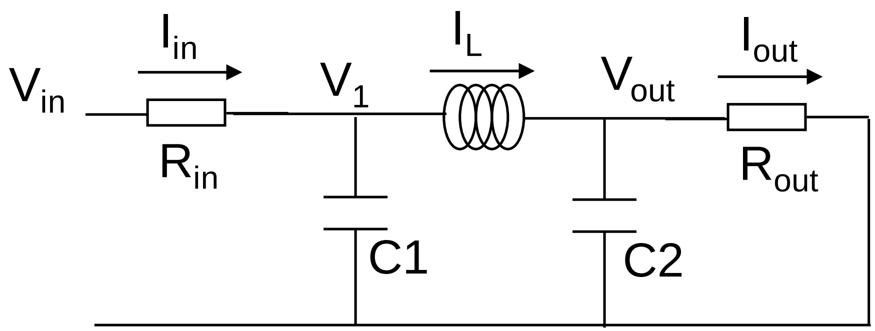
4. Map the biquad circuit element values to the biquad coefficients in the s-domain representation.

5. Scale R and C to meet noise spec. Adjust circuit bias, etc. to meet linearity.

Note: try not to exceed a biquad Q more than 5, else Q tuning will be needed.

Ladder Filter implementation using Opamp-RC

- SFG realization using opamp-RC



KCL/KVL:

- *Voltage nodes as a function of other voltage nodes and series branches impedance times their currents*
- *Node currents as a function of other currents and shunt branches admittance times their voltages*

$$V_1 = -R_{in} I_{in} + V_{in}$$
$$V_{out} = -sLI_L + V_1$$
$$0 = -R_{out} I_{out} + V_{out}$$

Use scaling resistor R for currents**

$\longrightarrow$

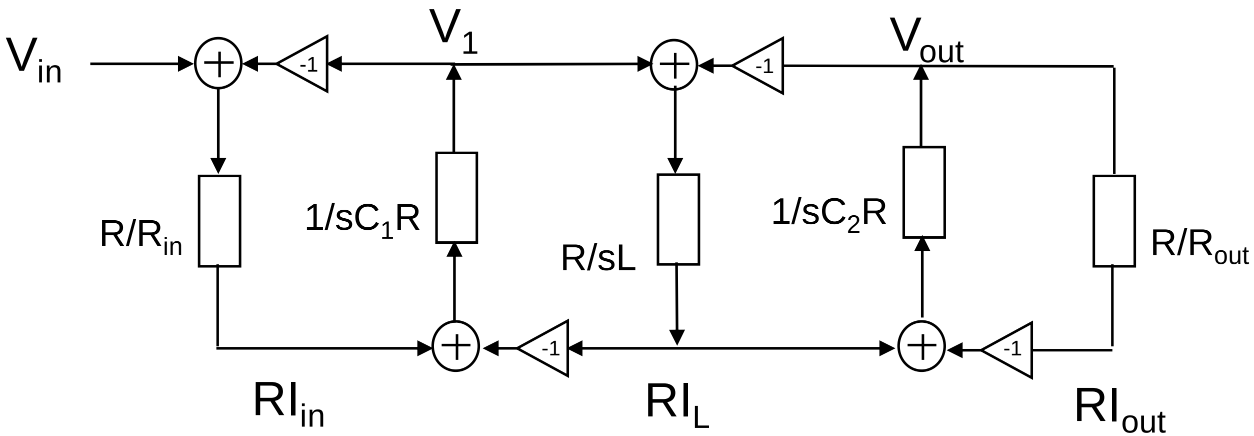
$$V_1 = -R_{in} \frac{R}{R} I_{in} + V_{in}$$
$$V_{out} = -sL \frac{R}{R} I_L + V_1$$
$$0 = -\frac{R}{R} R_{out} I_{out} + V_{out}$$

$$I_{in} = sC_1 V_1 + I_L$$
$$I_L = sC_2 V_{out} + I_{out}$$

$$RI_{in} = sC_1 RV_1 + RI_L$$
$$RI_L = sC_2 RV_{out} + RI_{out}$$

** *opamp RC circuits work with voltages, not currents*

Construct the signal-flow graph (SGF):



$$V_1 = -R_{in}\frac{R}{R}I_{in} + V_{in} \Rightarrow \left(V_{in} - V_1\right)\frac{R}{R_{in}} = RI_{in}$$

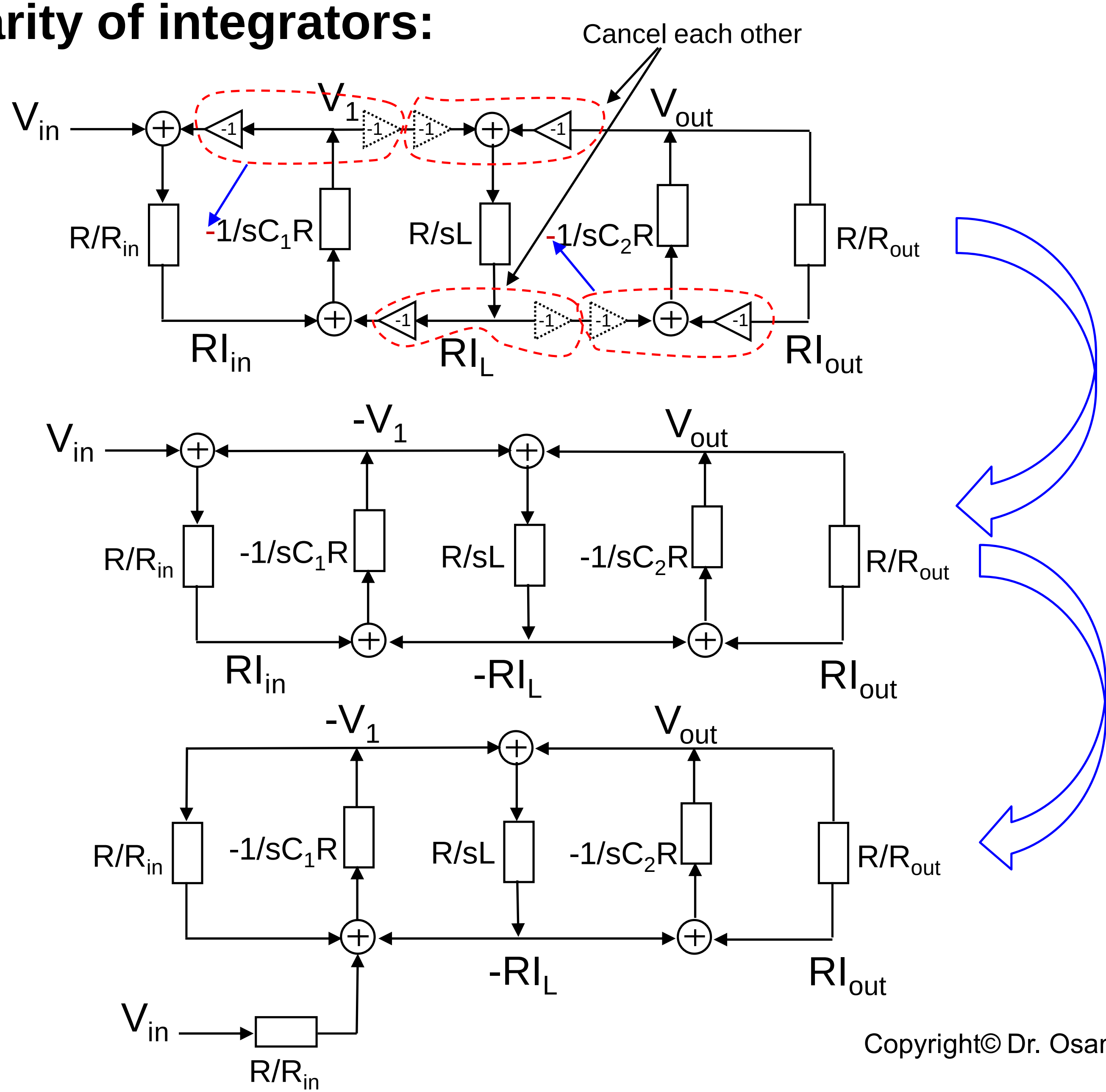
$$V_{out} = -sL\frac{R}{R}I_L + V_1 \Rightarrow \left(V_1 - V_{out}\right)\frac{R}{sL} = RI_L$$

$$0 = -\frac{R}{R}R_{out}I_{out} + V_{out} \Rightarrow \frac{R}{R_{out}}V_{out} = RI_{out}$$

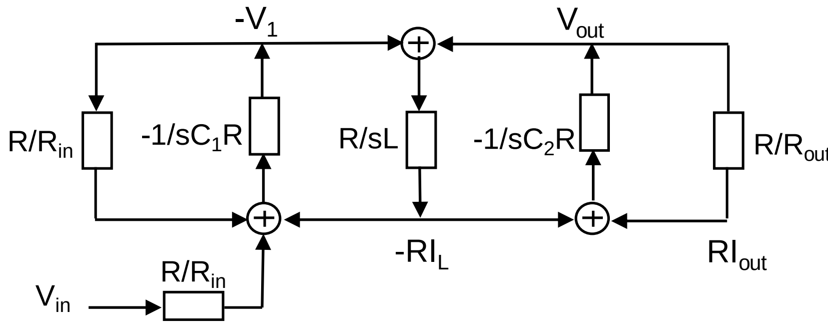
$$RI_{in} = sC_1RV_1 + RI_L \Rightarrow \frac{1}{sC_1R}\left(RI_{in} - RI_L\right) = V_1$$

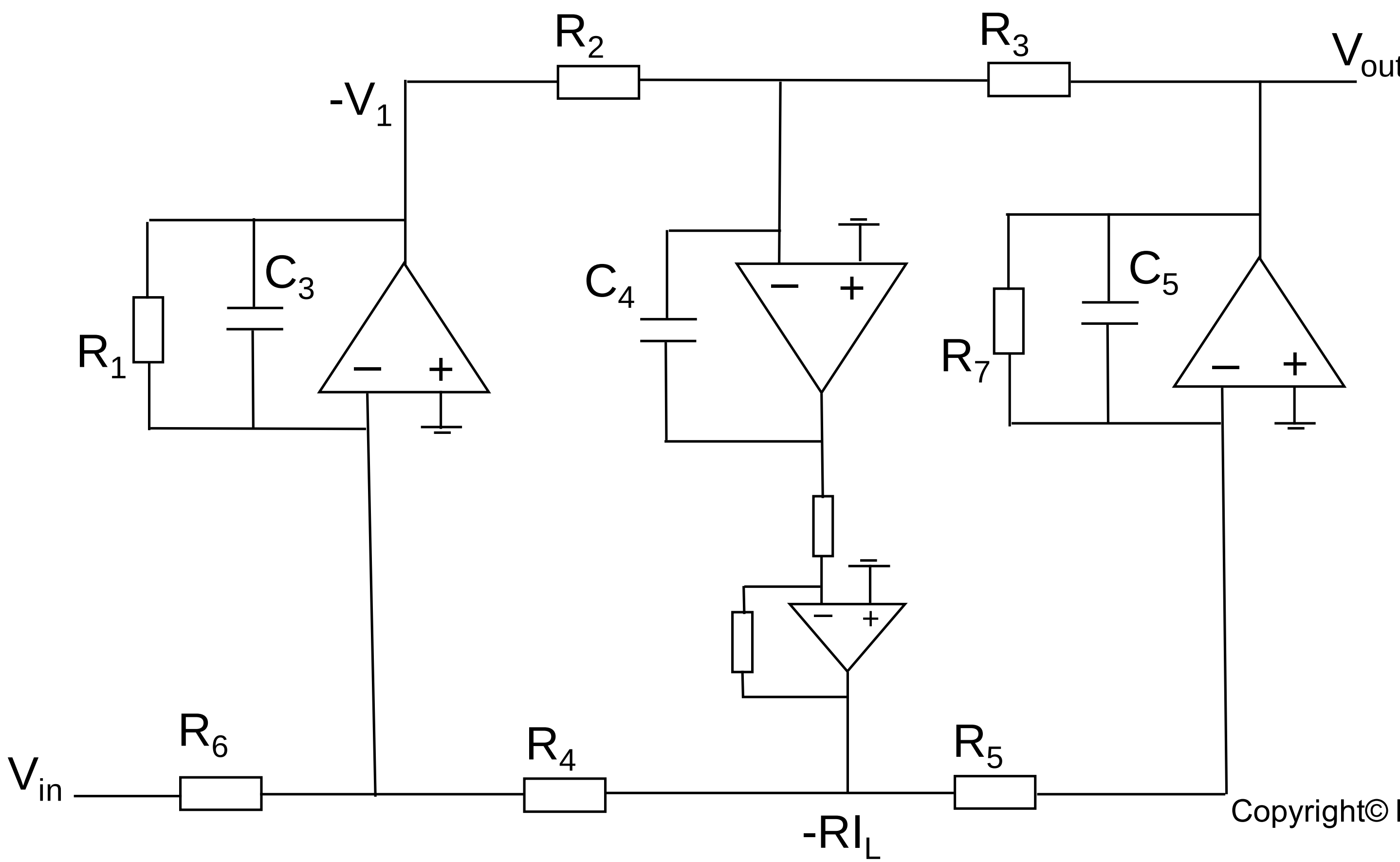
$$RI_L = sC_2RV_{out} + RI_{out} \Rightarrow \frac{1}{sC_2R}\left(RI_L - RI_{out}\right) = V_{out}$$

Set polarity of integrators:



Ladder filter opamp-RC realization:





Compare SFG to opamp-RC realization:

From opamp circuit:

$$V_{in} \rightarrow -V_1 : -\frac{1}{R_6}\frac{1}{sC_3}$$

$$-V_1 \rightarrow -V_1 : -\frac{1}{R_1}\frac{1}{sC_3}$$

$$-V_1 \rightarrow -RI_L : \frac{1}{R_2}\frac{1}{sC_4}$$

$$-RI_L \rightarrow -V_1 : \frac{1}{R_4}\frac{1}{sC_3}$$

$$V_{out} \rightarrow -RI_L : \frac{1}{R_3}\frac{1}{sC_4}$$

$$-RI_L \rightarrow V_{out} : \frac{1}{R_5}\frac{1}{sC_5}$$

$$V_{out} \rightarrow V_{out} : \frac{1}{R_7}\frac{1}{sC_5}$$

From SFG:

$$V_{in} \rightarrow -V_1 : -\frac{R}{R_{in}}\frac{1}{sC_1R} = -\frac{1}{sC_1R_{in}}$$

$$-V_1 \rightarrow -V_1 : -\frac{R}{R_{in}}\frac{1}{sC_1R} = -\frac{1}{sC_1R_{in}}$$

$$-V_1 \rightarrow -RI_L : \frac{R}{sL}$$

$$-RI_L \rightarrow -V_1 : \frac{1}{sC_1R}$$

$$V_{out} \rightarrow -RI_L : \frac{R}{sL}$$

$$-RI_L \rightarrow V_{out} : -\frac{1}{sC_2R}$$

$$V_{out} \rightarrow V_{out} : -\frac{R}{R_{out}}\frac{1}{sC_2R} = -\frac{1}{sC_2R_{out}}$$

conclusion:

$$C_3 = C_1 \quad , \quad R_6 = R_{in}$$

$$R_1 = R_{in}$$

$$R_2C_4 = \frac{L}{R}$$

$$R_4 = R$$

$$R_3C_4 = \frac{L}{R}$$

$$C_5 = C_2 \quad , \quad R_5 = R$$

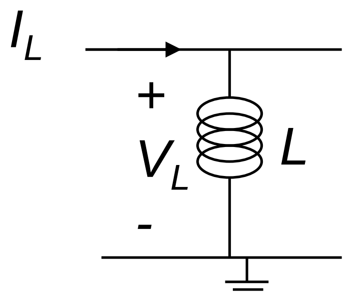
$$R_7 = R_{out}$$

The unknowns are C_4, (R_2, R_3), and (R_4,R_5). Set a value for R and C4 and calculate R2, R3, R4 and R5.

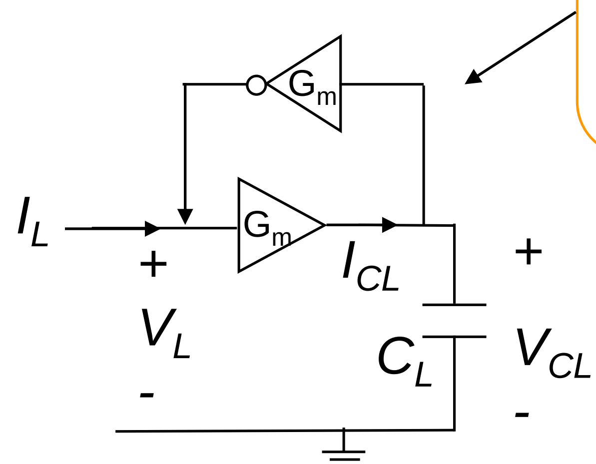
Filter implementation using Gm-C

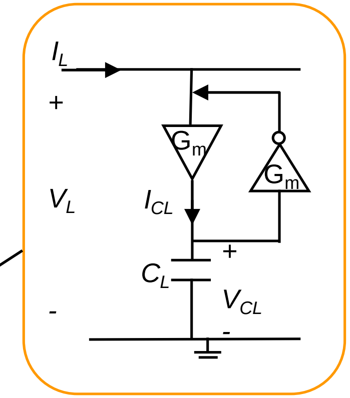
How to implement and “active” inductor?

- Grounded inductor:



$$V_L = sLI_L$$



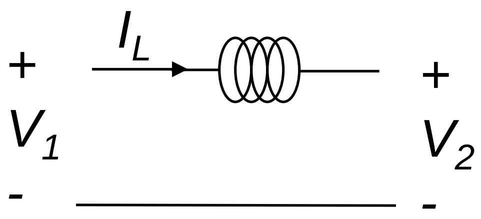


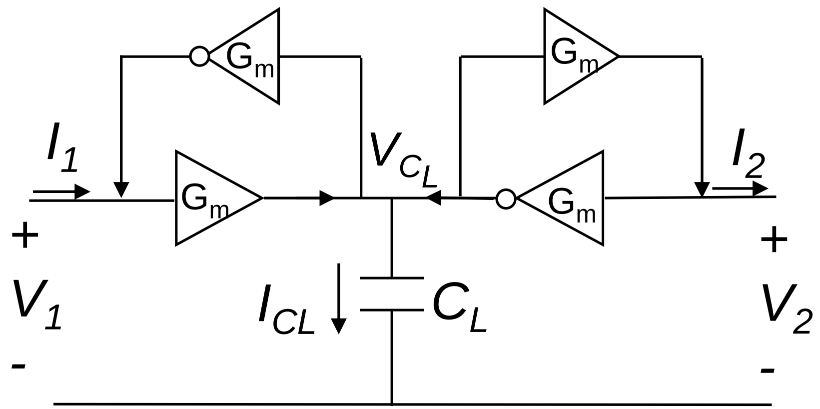
$$I_L = G_m V_{C_L}$$

$$V_{C_L} = \frac{1}{sC_L} I_{C_L} = \frac{G_m}{sC_L} V_L$$

$$\Rightarrow V_L = \frac{sC_L}{G_m^2} I_L \quad ; \text{and} \quad L = \frac{C_L}{G_m^2}$$

- Floating inductor:





$$I_L = \frac{1}{sL}(V_1 - V_2)$$

$$I_{C_L} = G_m(V_1 - V_2)$$

$$V_{C_L} = \frac{1}{sC_L} I_{C_L} = \frac{G_m}{sC_L}(V_1 - V_2)$$

$$\Rightarrow I_1 = G_m V_{C_L} = \frac{G_m^2}{sC_L}(V_1 - V_2)$$

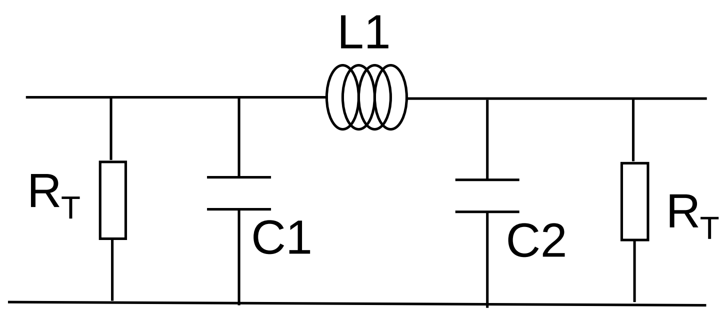
$$I_2 = G_m V_{C_L} = \frac{G_m^2}{sC_L}(V_1 - V_2) = I_1$$

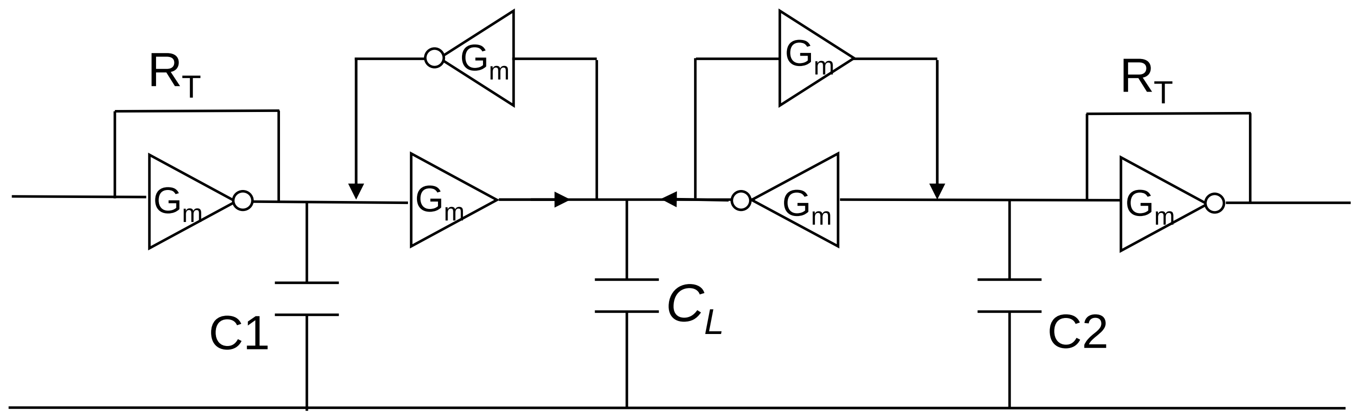
$$\Rightarrow L = \frac{C_L}{G_m^2}$$

The active Gm-C implementation of an inductor is sometimes called a "gyrator"

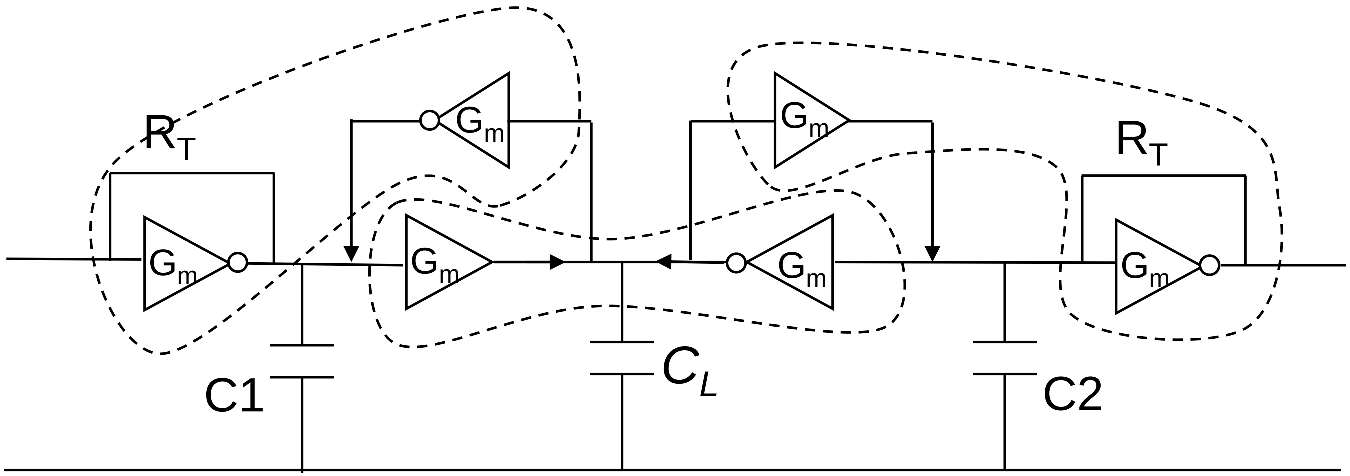
Ladder Gm-C implementation: direct element replacement

Just replace inductors of the LC ladder with active equivalent



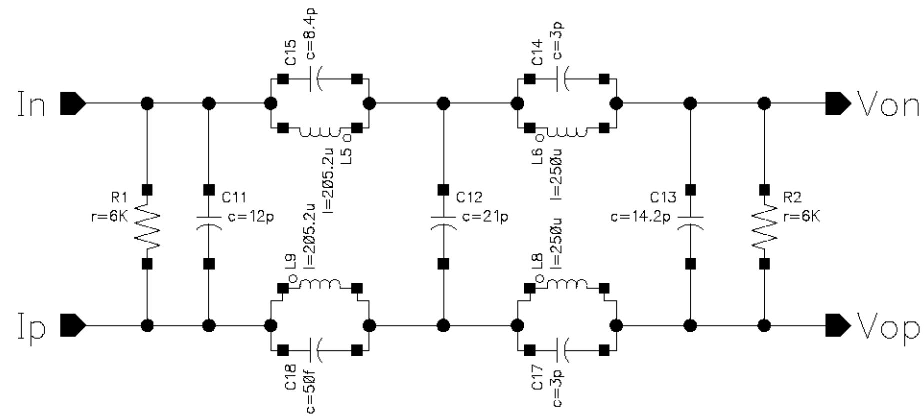


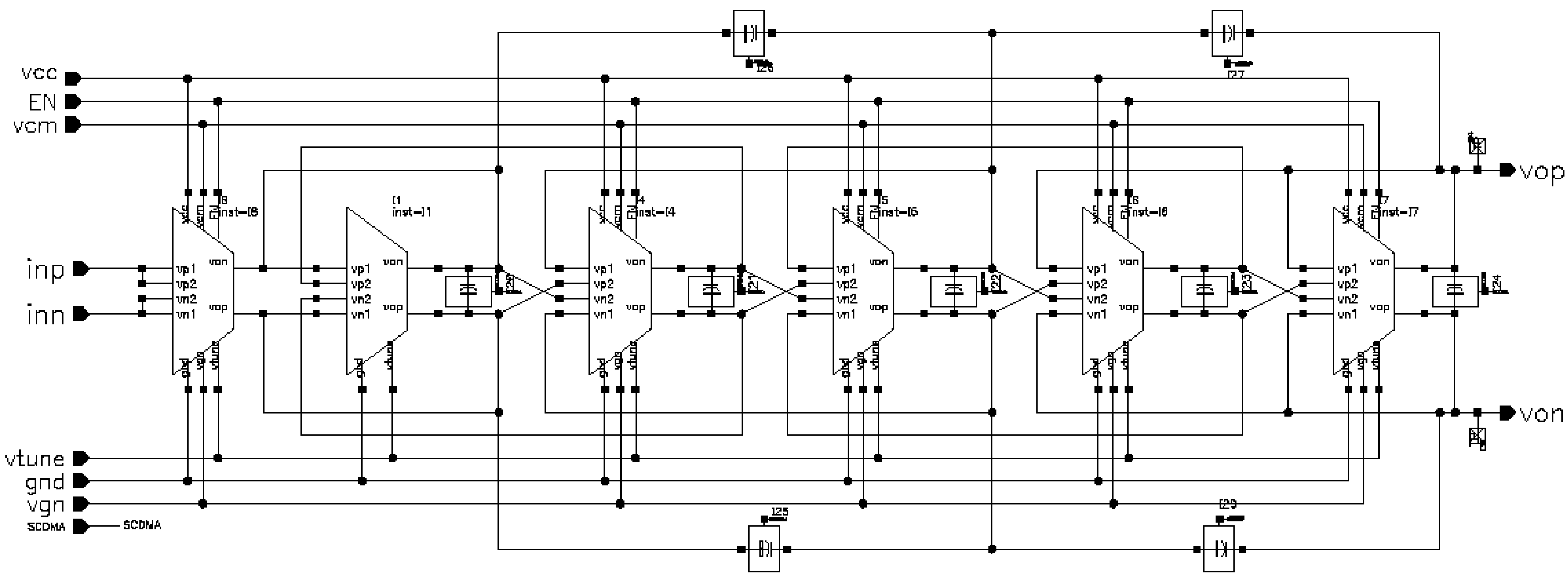
Differential Gm cells with their high impedance output require a common-mode feedback circuit to set the output common-mode DC voltage. Gm cells sharing the same output can be combined into multi-input single-output Gm cell with one common-mode circuit.



Example: 5^{th} order elliptic Gm-C lowpass filter

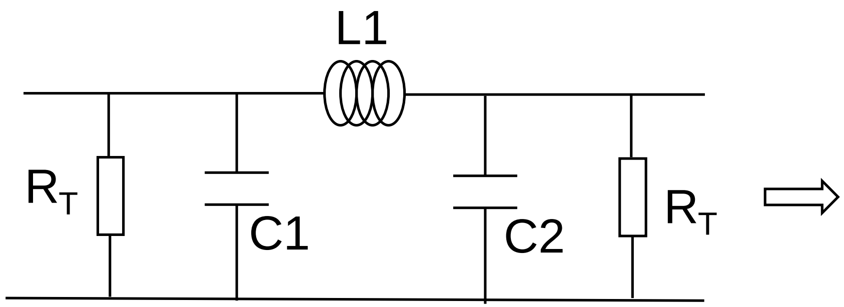
Equivalent LC representation →

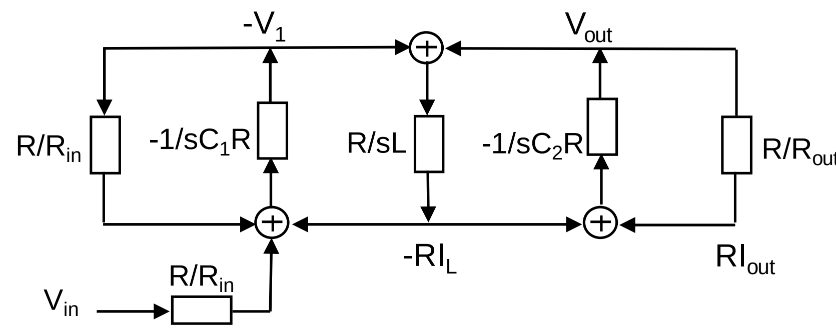


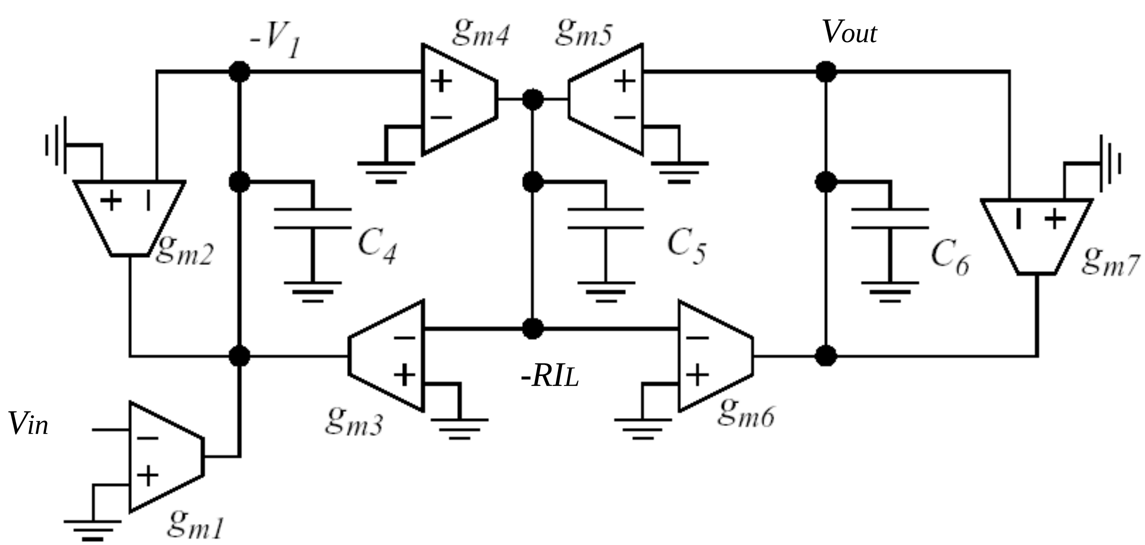


Ladder Filter implementation using Gm-C

Follow same SFG we did previously to realize the Gm-C ladder. Gm cell outputs can be directly connected together for the sum:







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[1] R. Schaumann, K. Laker, M. Ghaussi, “Design of Analog Filters, Passive, Active RC, and Switched Capacitor,” Prentice Hall, 1990