Complex Filters. Filter parasitics and Tuning

- **Filter Tuning**
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  - Tuner architectures
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- **Tuning:**

Why tuning?

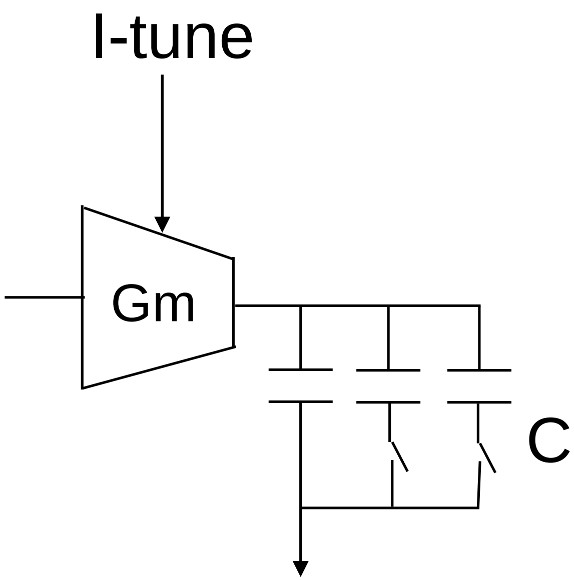
- ω is in rad/sec so it must depend on absolute value of circuit elements (R, C, Gm).
- circuit elements are subject to variation due to process and temp.

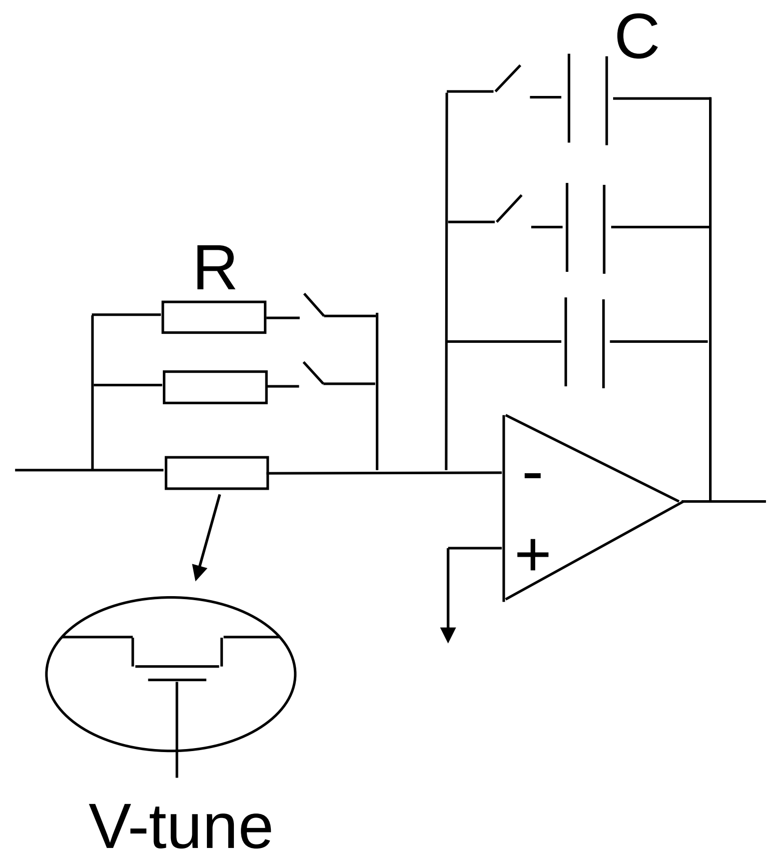
$\Rightarrow$ an integrated filter MUST be tuned

- filter Q is dimensionless, it depends on the ratio of like circuit elements.

$\Rightarrow$ Q needs to be tuned only for high-Q filters (>6)

Tuning methods:



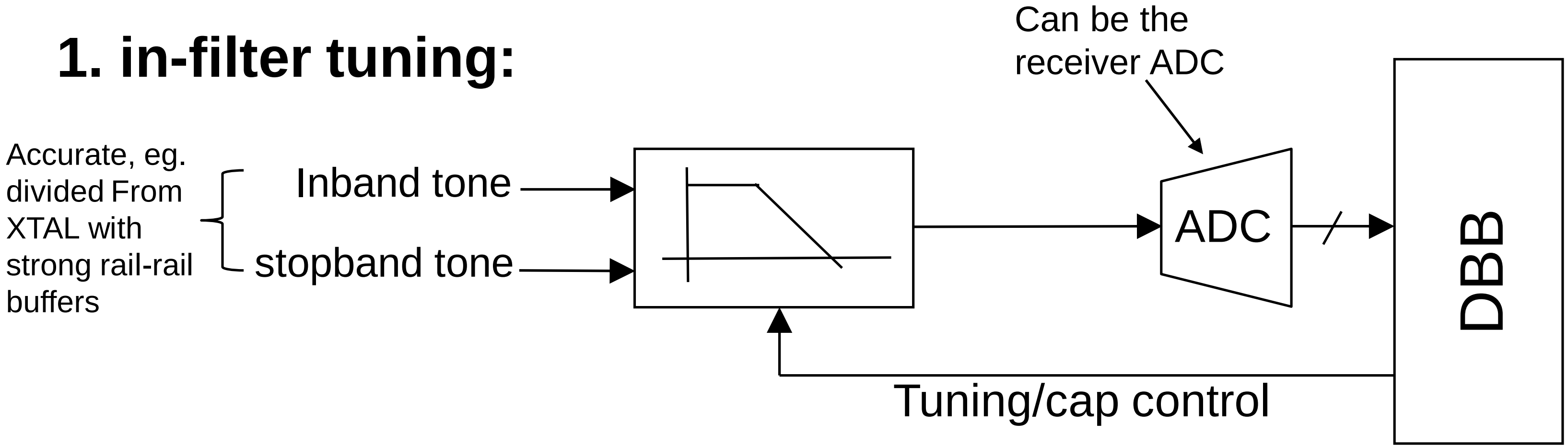


- Digital tuning adds to area overhead. It also introduces parasitic poles/zero to the overall TF.

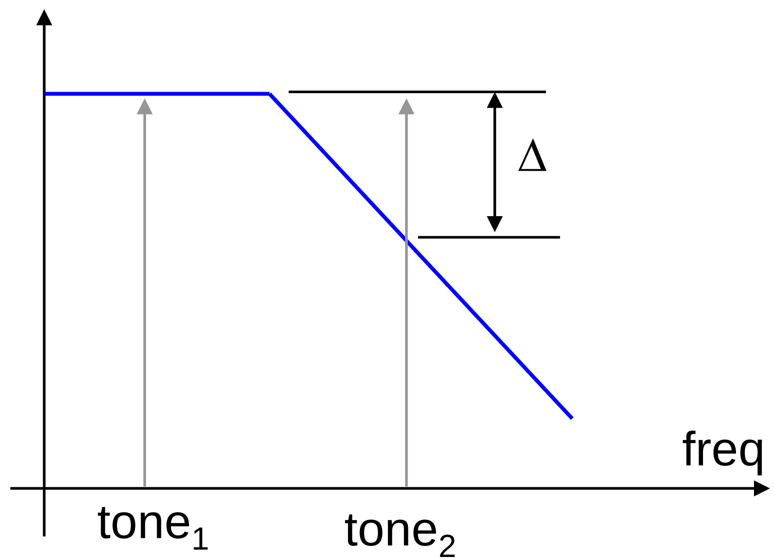
- Analog tuning is more elegant. It affects linearity over process, temperature and Vcc

Common Filter tuning schemes:

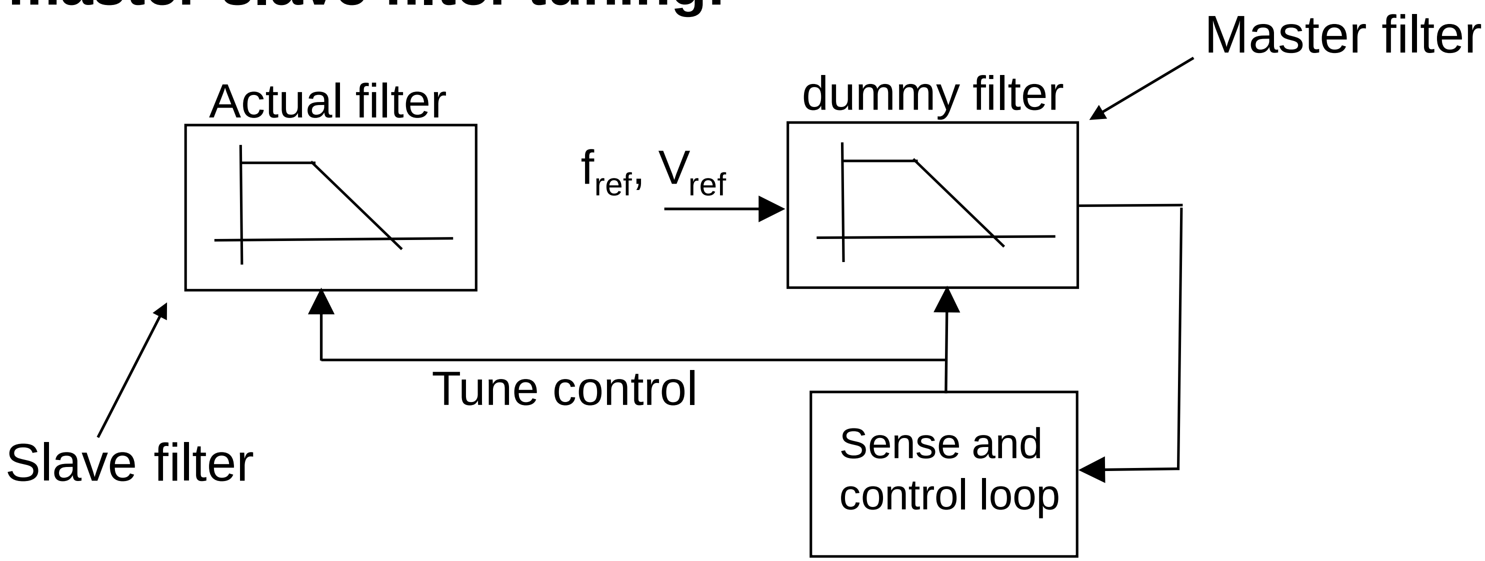
1. in-filter tuning:



Two equal tones are injected into the filter, one at a time. The first tone is at a passband frequency, while the other is at a stopband frequency with known filter rejection value. The two tones are measured at the filter output with the aid of the ADC. Digital baseband adjusts the filter tuning code until the exact rejection value is met. This assumes the filter is only process dependent, but temperature and Vdd independent. Retuning is needed when temperature drifts

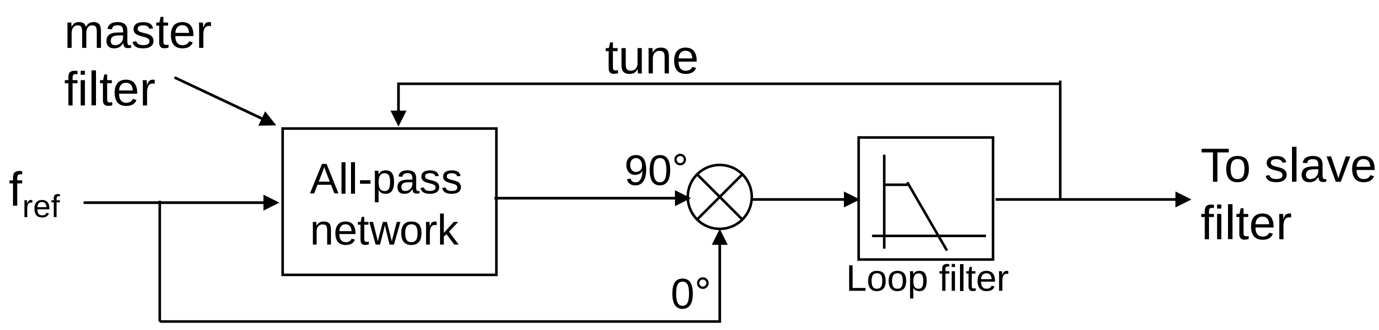


2. master-slave filter tuning:



The slave filter is the actual filter to be tuned. The master filter is a cell representing the filter, which can be a simple opam-RC or Gm/C unit or an entire biquad of same component types used to construct the slave filter. A precision voltage reference (bandgap), or frequency clock (Xtal oscillator) is injected into the master filter. An output dependent on the RC product or Gm/C ratio is monitored. A feedback loop changes R's, C's or both (Gm also) to lock the loop. The same tune word/signal used to tune/lock the master filter loop is used directly to tune the slave filter.

An example of a master/slave filter tuning scheme:

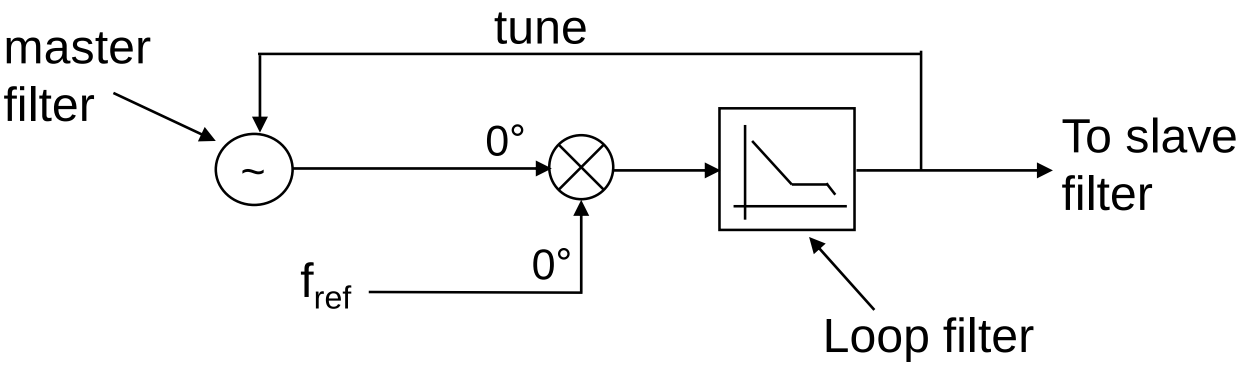


The allpass network uses the same Gm/C or opamp-RC blocks in the main filter to construct the following function:

$$H(s)_{allpass} = \frac{s - P_0}{s + P_0}$$

The magnitude of the allpass network is unity, but the phase is 90° at $\omega = P_0$. The allpass pole/zero, P_0, is set to depend on Gm/C, or opamp-RC, just like the main filter 3dB frequency. If the process is well centered, the output of the phase detector is zero. If, however, the process is not centered, the output of the phase detector is finite and the feedback control signal will adjust (tune) the all pass network to force the value of P_0 to equal the nominal value. Note that for this to work, a precision reference clock (Xtal) is needed whose oscillation frequency is P_0.

Another example of a master-slave filter tuning scheme:

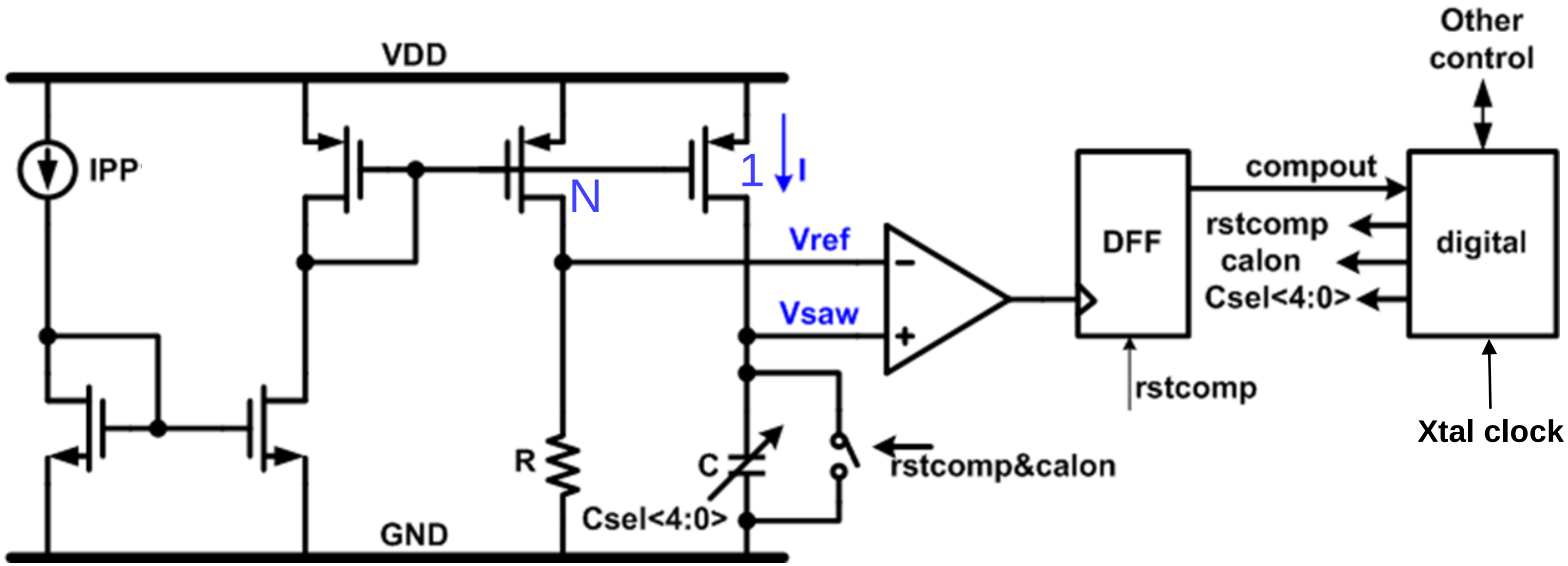


The above architecture is a simple phase-locked loop. The VCO is constructed using the same Gm/C or opamp-RC cells used in the main filter. This is a relaxation-type VCO whose oscillation frequency ω_0 is dependent on Gm/C or RC. The loop ensures that the VCO frequency is always equal to the precision reference frequency f_{ref} which is usually coming from a xtal oscillator.

A VCO can be constructed by putting two lossless integrators in a feedback loop with some loss compensation for startup loop gain.

Note that for practical reasons, the reference frequency is used way out of band to prevent clock feed through. Moreover, the tune signal is heavily filtered before feeding it to the slave (main) filter.

Automatic Filter tuning scheme based on RC time constant:



Initially cap is reset (discharged). When rstcomp signal opens the switch, the cap C start charging and the digital starts counting Xtal clocks. When Vsaw reaches Vref, comparator output goes high and sends a signal to digital to stop counting.

$$V_{ref} = N.I.R = N\frac{V_{BG}}{R_{ref}}R = costant$$

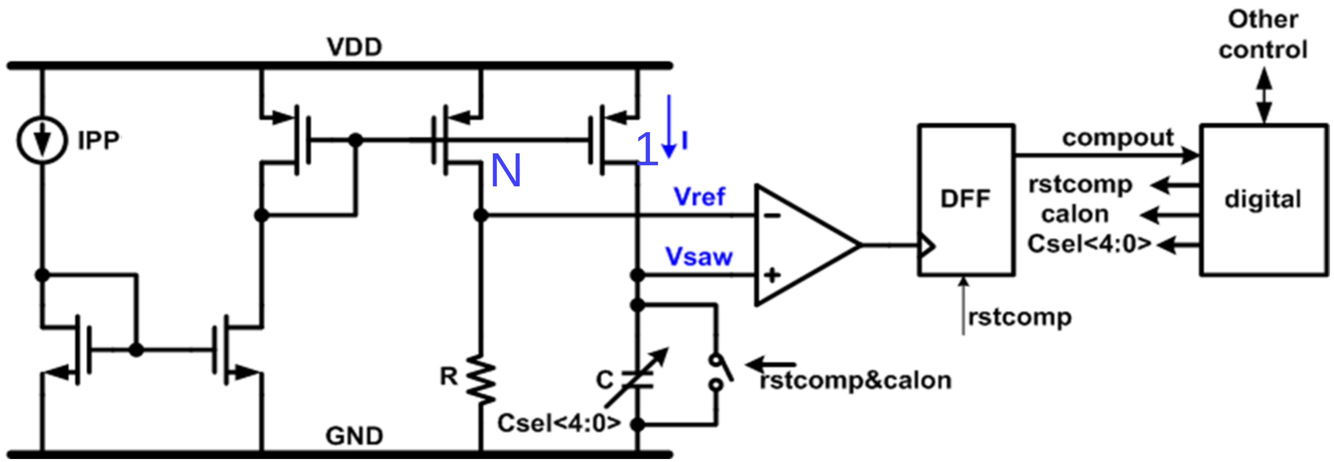
$$V_{saw} = \frac{1}{C}\int I\,dt = \frac{V_{BG}}{R_{ref}}\frac{1}{C}T_{saw}\ ,$$

where T_{saw} is the time it takes for the voltage of the fully discharged cap C to reach V_{ref}

$$\rightarrow N\frac{V_{BG}}{R_{ref}}R = \frac{V_{BG}}{R_{ref}}\frac{1}{C}T_{saw}$$

$$\rightarrow \boldsymbol{T_{saw} = N.RC}$$

Automatic Filter tuning based on RC time constant, Cont':



→ $\boldsymbol{T_{saw} = N.RC}$

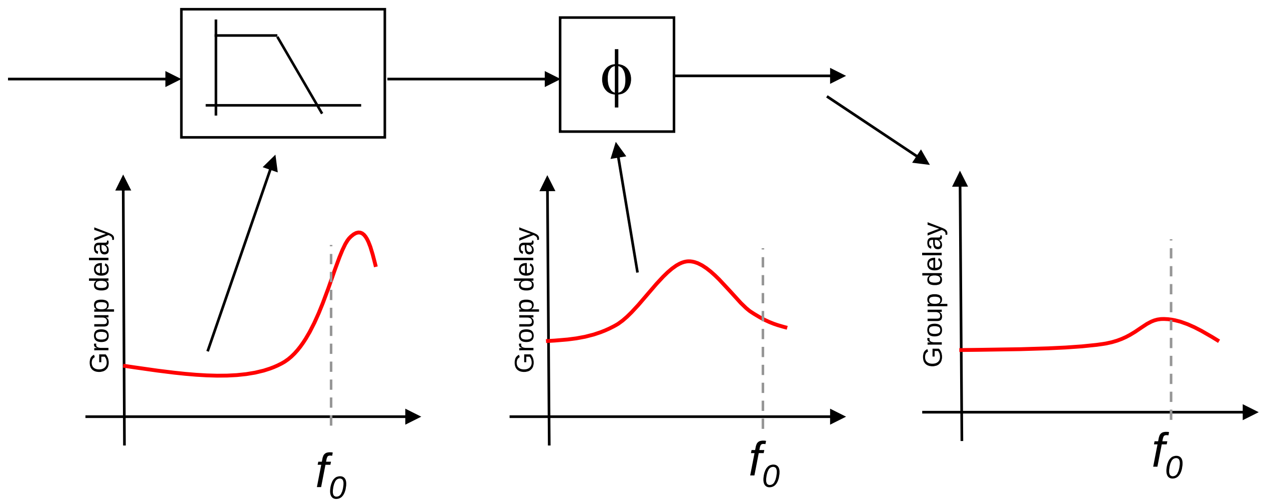
If the precise reference clock (from crystal oscillator) has a period of T_{ref}, with $T_{ref} << T_{saw}$, we can then write T_{saw} as M number of reference clock periods as:

$$T_{saw} = M.T_{ref}$$

$$\rightarrow RC = \frac{M}{N} T_{ref}$$

So, for a given nominal value of *RC,* there is an exact number of clock periods *M* that need to be counted during the calibration cycle (stored in the chip memory). If the either (or both) of R or C values is off nominal, the tuning cap bank of C (<4:0> in the example above) is adjusted until the count *M* is met. The resulting cap bank code is then sent to all cap banks in the actual filter which also uses <4:0> to match the tuning circuit.

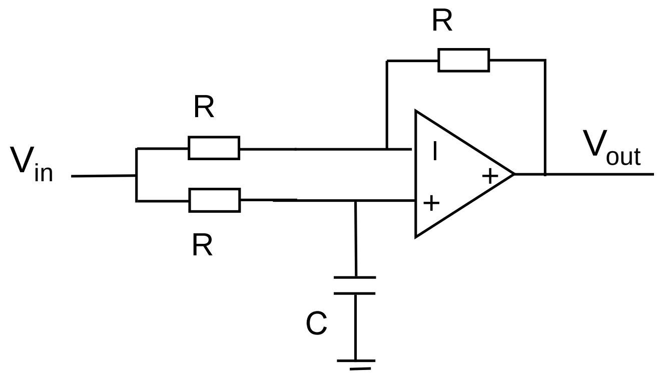
Group delay equalizers:



Allpass networks are used for filter group delay equalization. This is because they only impact the phase (and hence group delay) of the filter and not the filter amplitude.

Sometimes multiple all-pass networks are used to get the desired group delay equalization. The order of the group delay equalizer equals the order of the allpass function in the s-domain.

Example of a differential group-delay equalizer:



$$H(s) = \frac{s - z_0}{s + p_0} \quad ; \ |z_0| = |p_0|$$

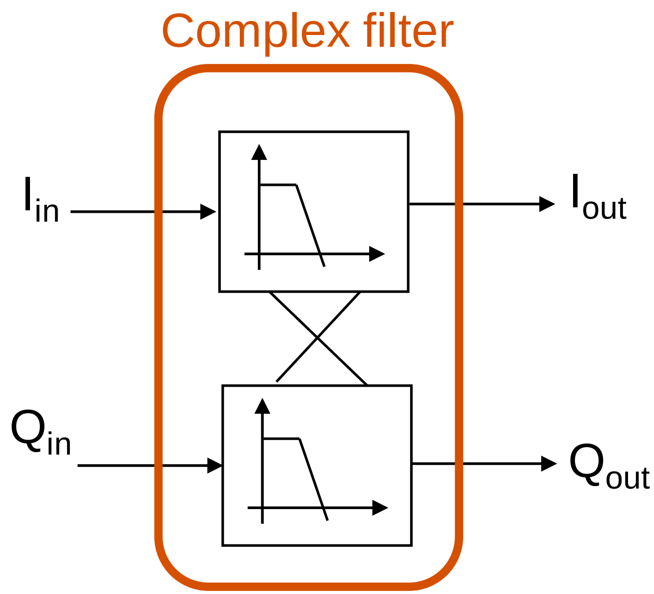
$$H(s) = -\frac{s - 1/(RC)}{s + 1/(RC)}$$

Equalizer gain is flat over frequency but its phase is that of two concentric real poles (90-degrees phase shift at pole freq)

The allpass network does not have to directly follow the baseband filter. It can be placed anywhere in the baseband chain. For noise figure purposes, it is best to place group delay equalizers after the first stage VGA.

Complex active filters:

What is a complex filter?

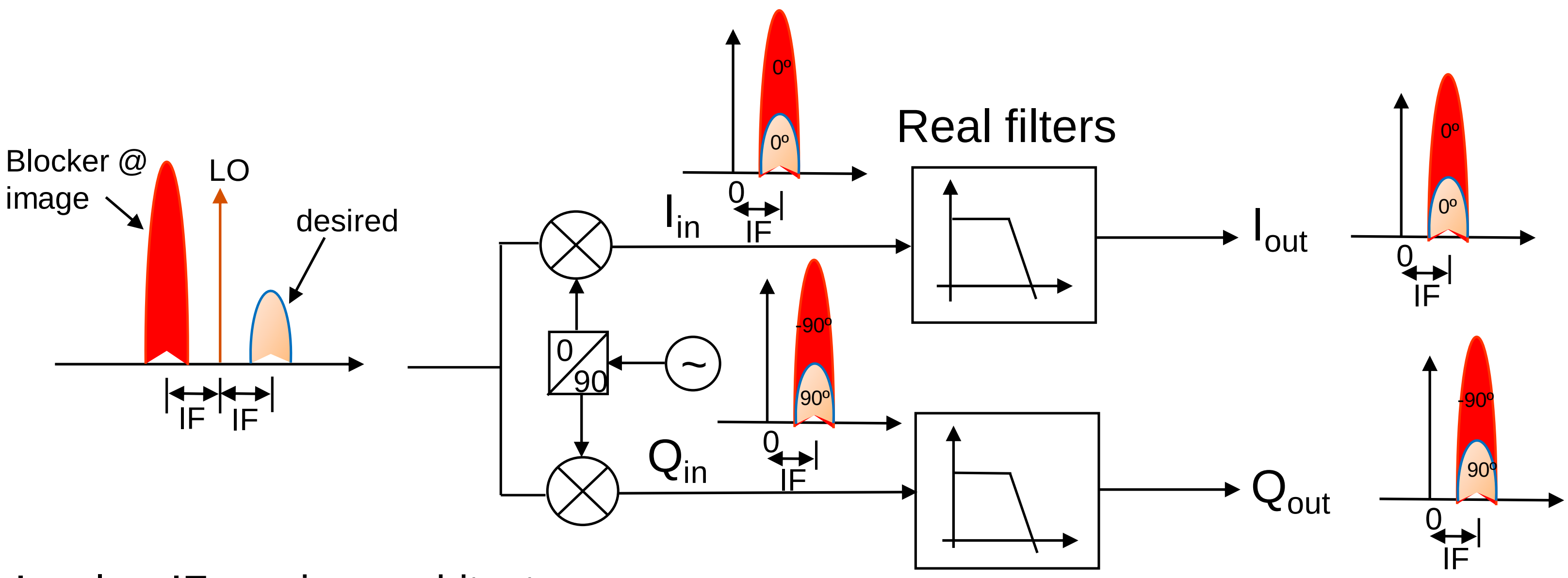


There is nothing complex about complex filters. They are called "complex" because they have two orthogonal signals (I and Q) as inputs and each of their orthogonal (complex) outputs depends on both complex inputs. Mathematically, it is easier to represent orthogonal signals as complex signals (I + jQ). This representation makes math a lot easier. For example, if

$$I = \cos(\omega_0 t) \quad ;\text{and} \quad Q = \sin(\omega_0 t)$$

$$\Rightarrow input = I + jQ = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

where complex filter is best used?



In a low-IF receiver architecture:

$$RF = m_r(t)\cos(\omega_{LO}+\omega_{IF})t + m_I(t)\cos(\omega_{LO}-\omega_{IF})t$$

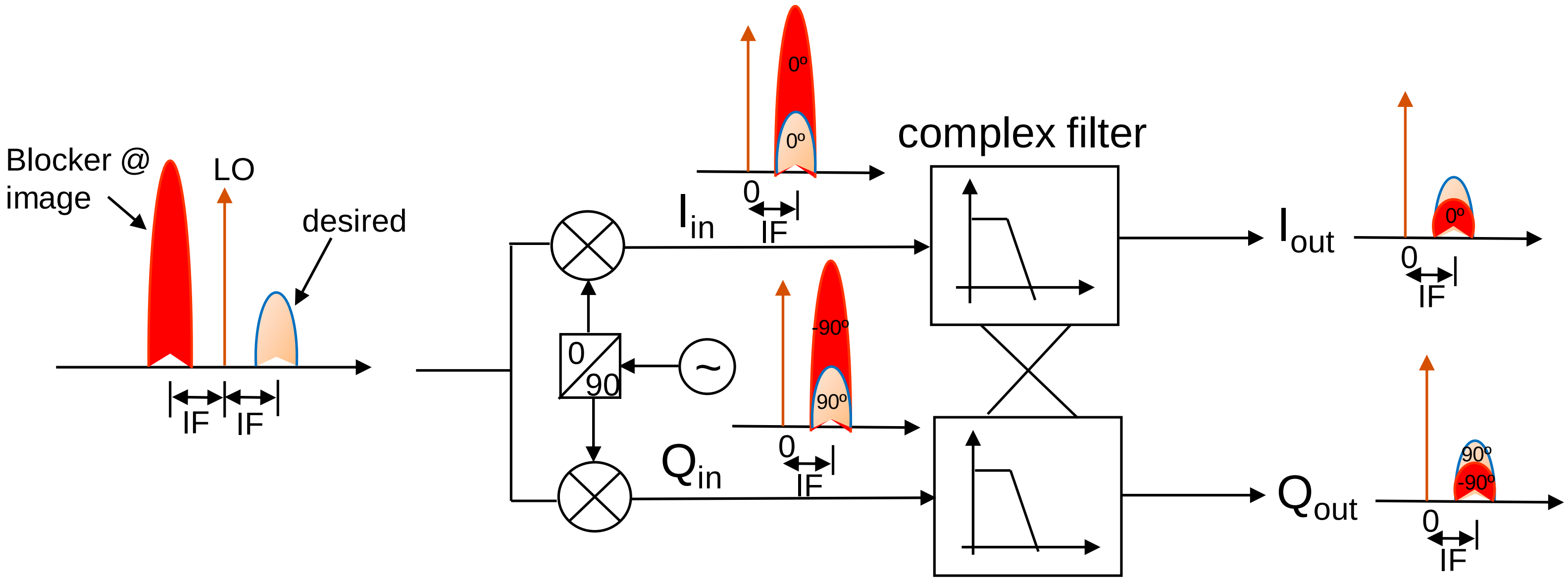
image

$$I_{in} = RF \times \cos(\omega_{LO}t) = \frac{1}{2}\left(m_r(t)\cos(\omega_{IF})t + m_I(t)\cos(-\omega_{IF})t\right)$$

$$Q_{in} = RF \times \sin(\omega_{LO}t) = \frac{1}{2}\left(m_r(t)\sin(\omega_{IF})t + m_I(t)\sin(-\omega_{IF})t\right)$$

As seen the image blocker (can be 10'sdB higher than desired) falls on top of desired signal. Real low-pass filter does nothing to this because blocker already fell in-band. The ADC then needs to have enough dynamic range to handle this combined power of blocker+desired so digital baseband can perform image suppression.

where complex filter is best used?



In a low-IF receiver architecture:

$$RF = m_r(t)\cos(\omega_{LO}+\omega_{IF})t + m_I(t)\cos(\omega_{LO}-\omega_{IF})t$$

image

$$I_{in} = RF\times\cos(\omega_{LO}t) = \frac{1}{2}\left(m_r(t)\cos(\omega_{IF})t + m_I(t)\cos(-\omega_{IF})t\right)$$

$$Q_{in} = RF\times\sin(\omega_{LO}t) = \frac{1}{2}\left(m_r(t)\sin(\omega_{IF})t + m_I(t)\sin(-\omega_{IF})t\right)$$

A complex filter takes these two complex inputs and ideally passes the desired signal and "partially" suppresses the image; that is

$$I_{out} = \frac{1}{2}\left(m_r(t)\cos(\omega_{IF})t + \delta m_I(t)\cos(-\omega_{IF})t\right) \text{ ; and } Q_{out} = \frac{1}{2}\left(m_r(t)\sin(\omega_{IF})t + \delta m_I(t)\sin(-\omega_{IF})t\right)$$

Where δ is the complex filter image rejection factor

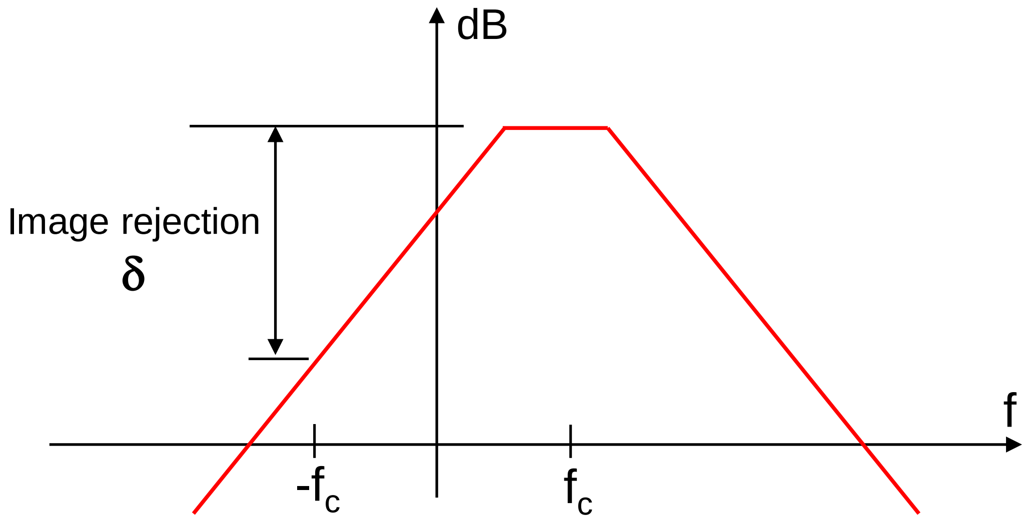
$$RF = m_r(t)\cos(\omega_{LO} + \omega_{IF})t + m_I(t)\cos(\omega_{LO} - \omega_{IF})t$$

negative freq

$$I_{in} = RF \times \cos(\omega_{LO} t) = \frac{1}{2}\left(m_r(t)\cos(\omega_{IF})t + m_I(t)\cos(-\omega_{IF})t\right)$$

$$Q_{in} = RF \times \sin(\omega_{LO} t) = \frac{1}{2}\left(m_r(t)\sin(\omega_{IF})t + m_I(t)\sin(-\omega_{IF})t\right)$$

It appears "as if" the complex filter discriminates against signals folded to baseband due to negative frequencies (means image frequency is smaller than LO resulting in negative IF freq). Therefore, the complex filter response looks like the following plot

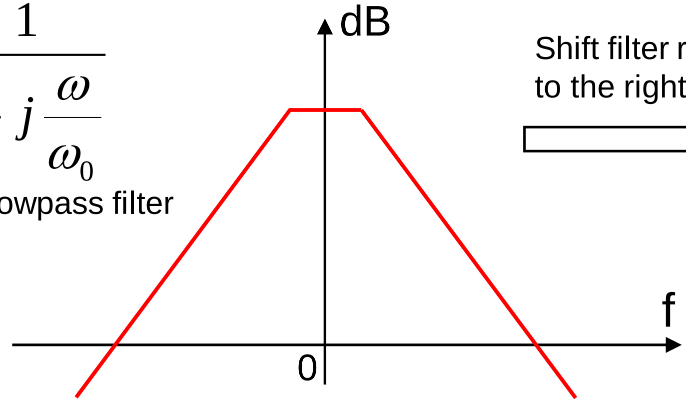


Note that the image rejection level, δ, is set by the filter order (3rd, 5th, etc.), filter function (Butterworth, Chebyshev, etc) and the frequency shift, f_c. It is also set by the I/Q input magnitude and phase matching. If achieved image rejection δ is sufficient for SNR, you need only either I or Q signal (not both) to send to digital baseband.

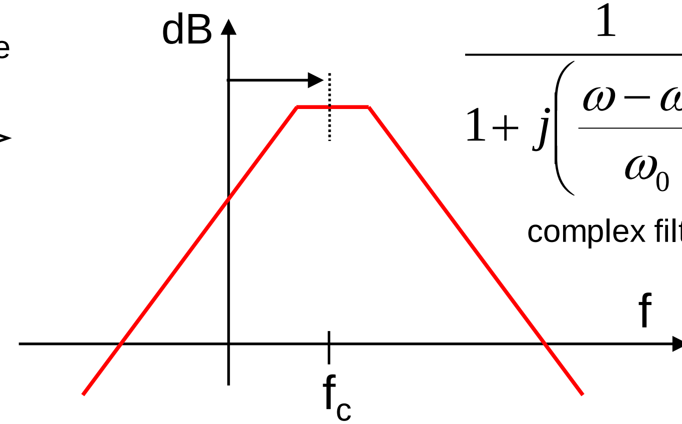
Conceptually, one would think that a way to build a complex filter is by starting with a real low pass filter and then somehow shift its response to the right by the desired bandpass center frequency.

$$\frac{1}{1+j\dfrac{\omega}{\omega_0}}$$

Real lowpass filter



Shift filter response to the right by f_c



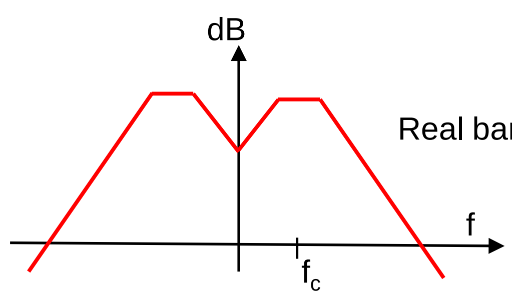
$$\frac{1}{1+j\left(\dfrac{\omega-\omega_C}{\omega_0}\right)}$$

complex filter

Complex filters in a sense show a simple way of realizing bandpass filters. However, there is a real difference between the two. Real bandpass filter does not discriminate against image.

This “complex” bandpass filter realization is compared to “real” bandpass filter as follows. A second order bandpass filter transfer function can be written as

$$H_{BP}(s)=\frac{cs}{s^2+as+b}$$



Real bandpass filter

The realization is basically a universal biquad which needs to be copied twice one for I and the other for Q.

Let us investigate the “complex” realization of this bandpass filter. We will start first with the lowpass response, then shift the filter frequency response by ω_c, as follows

$$H_{LP}(j\omega)=\frac{1}{1+j\omega RC} \quad \rightarrow \quad H_{CBP}(j\omega)=H_{LP}(j\omega-j\omega_c)=\frac{1}{1+j(\omega-\omega_c)RC}$$

The complex filter will be realized through an integrator and an amplifier as follows

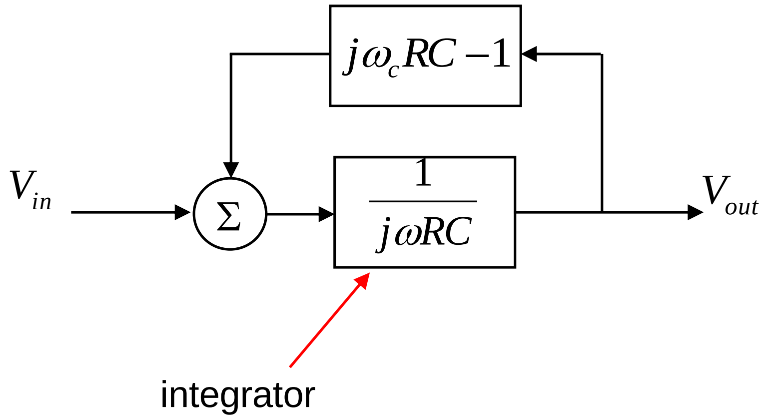
$$H_{CBP}(j\omega)=\frac{V_{out}}{V_{in}}=\frac{1}{1+j(\omega-\omega_c)RC}=\frac{1}{1+j\omega RC-j\omega_c RC} \quad ; \text{ both } V_{out} \text{ and } V_{in} \text{ are complex signals}$$

$$\Rightarrow V_{out}(1+j\omega RC-j\omega_c RC)=V_{in}$$

$$V_{out}(j\omega RC)=V_{in}+V_{out}(j\omega_c RC-1)$$

$$\Rightarrow V_{out}=\frac{1}{(j\omega RC)}\left[V_{in}+V_{out}(j\omega_c RC-1)\right]$$

The first order complex filter block diagram based on the above equation is:

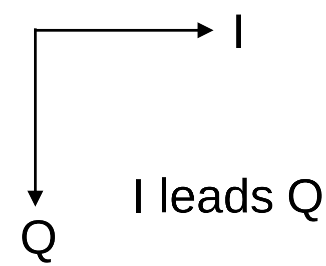


So for an I and Q signals with “I” leading “Q”:

$$V_{out\_I} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)}\left[V_{in\_I} + V_{out\_I}\left(\frac{j\omega_c}{\omega_0} - 1\right)\right] \quad ;\omega_0 = \frac{1}{RC}$$

if <u>I leads Q</u>, then j(I) = -Q (assuming I and Q are matched in amplitude and are 90-degrees out of phase. If this condition is not exactly met, complex filter performance is affected)

$$\rightarrow V_{out\_I} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)}\left[V_{in\_I} - \frac{\omega_c}{\omega_0}V_{out\_Q} - V_{out\_I}\right]$$



Similarly for Q with jQ = I

$$\rightarrow V_{out\_Q} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)}\left[V_{in\_Q} + \frac{\omega_c}{\omega_0}V_{out\_I} - V_{out\_Q}\right]$$

Therefore, the complex filter transfer function for outputs V_{out_I} (and V_{out_Q}) as a function of complex inputs V_{in_I} and V_{in_Q} becomes:

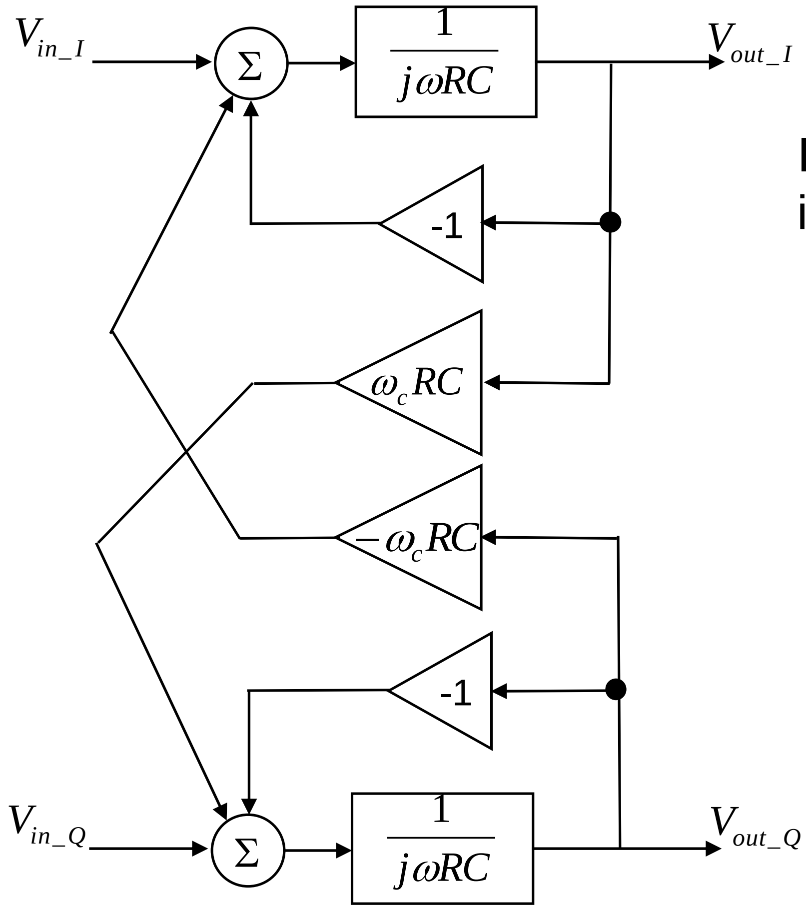
$$V_{out\_I} = \frac{\left(1+\frac{j\omega}{\omega_0}\right)\cdot V_{in,I} - \left(\frac{\omega_C}{\omega_0}\right)\cdot V_{in,Q}}{\left(1+\frac{j\omega}{\omega_0}+\frac{j\omega_C}{\omega_0}\right)\cdot\left(1+\frac{j\omega}{\omega_0}-\frac{j\omega_C}{\omega_0}\right)} \quad ;V_{out\_Q} = \frac{\left(\frac{\omega_C}{\omega_0}\right)\cdot V_{in,I} + \left(1+\frac{j\omega}{\omega_0}\right)\cdot V_{in,Q}}{\left(1+\frac{j\omega}{\omega_0}+\frac{j\omega_C}{\omega_0}\right)\cdot\left(1+\frac{j\omega}{\omega_0}-\frac{j\omega_C}{\omega_0}\right)}$$

The first order complex filter full realization with I/Q signals looks like:

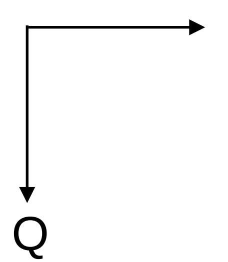
$$V_{out\_I} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)}\left[V_{in\_I} - \frac{\omega_c}{\omega_0}V_{out\_Q} - V_{out\_I}\right]$$

$$V_{out\_Q} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)}\left[V_{in\_Q} + \frac{\omega_c}{\omega_0}V_{out\_I} - V_{out\_Q}\right]$$

$$\begin{cases} \omega_C = 2\pi \cdot f_{IF} \\ \omega_0 = \dfrac{1}{RC} \end{cases}$$

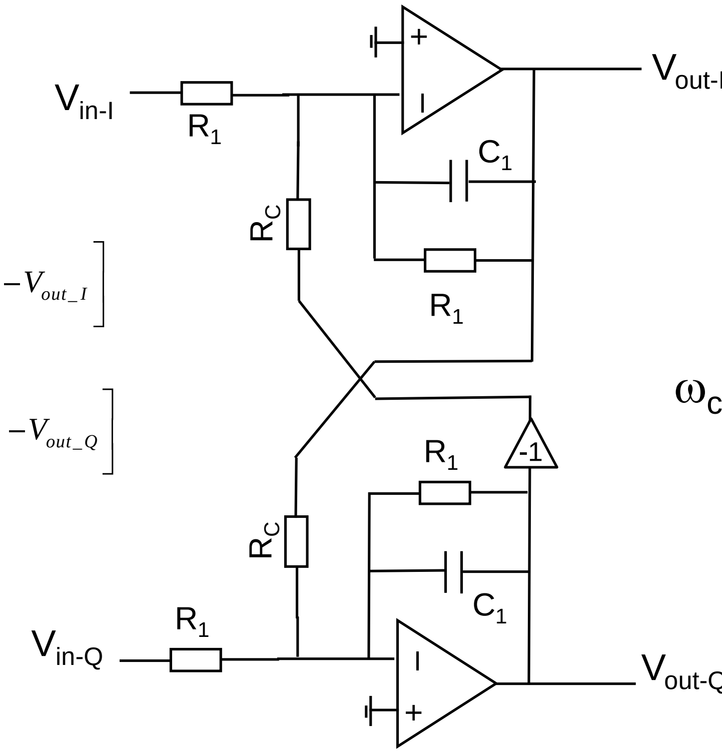


I leads Q in this implementation



Note that when ω_c is zero, the diagram collapses to a simple “real” first order lowpass filter for each I and Q.

The first order complex filter full realization with opamp-RC looks like:

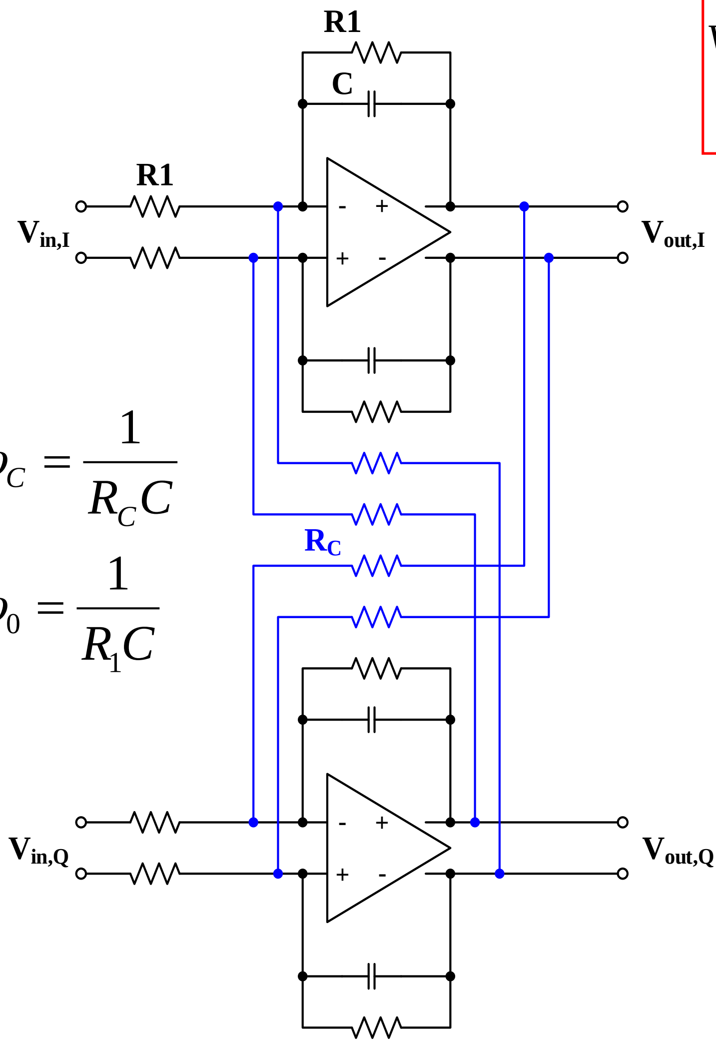


$$V_{out\_I} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)}\left[V_{in\_I} - \frac{\omega_c}{\omega_0}V_{out\_Q} - V_{out\_I}\right]$$

$$V_{out\_Q} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)}\left[V_{in\_Q} + \frac{\omega_c}{\omega_0}V_{out\_I} - V_{out\_Q}\right]$$

$\omega_c = 1/R_C C_1$, $\omega_0 = 1/R_1 C_1$

Note that the I/Q coupling resistor (R_C) decreases as the frequency shift ω_c increases. With ω_c =0 (no shift, $R_C=\infty$) the complex filter collapses to two independent I/Q real first-order lowpass filters.

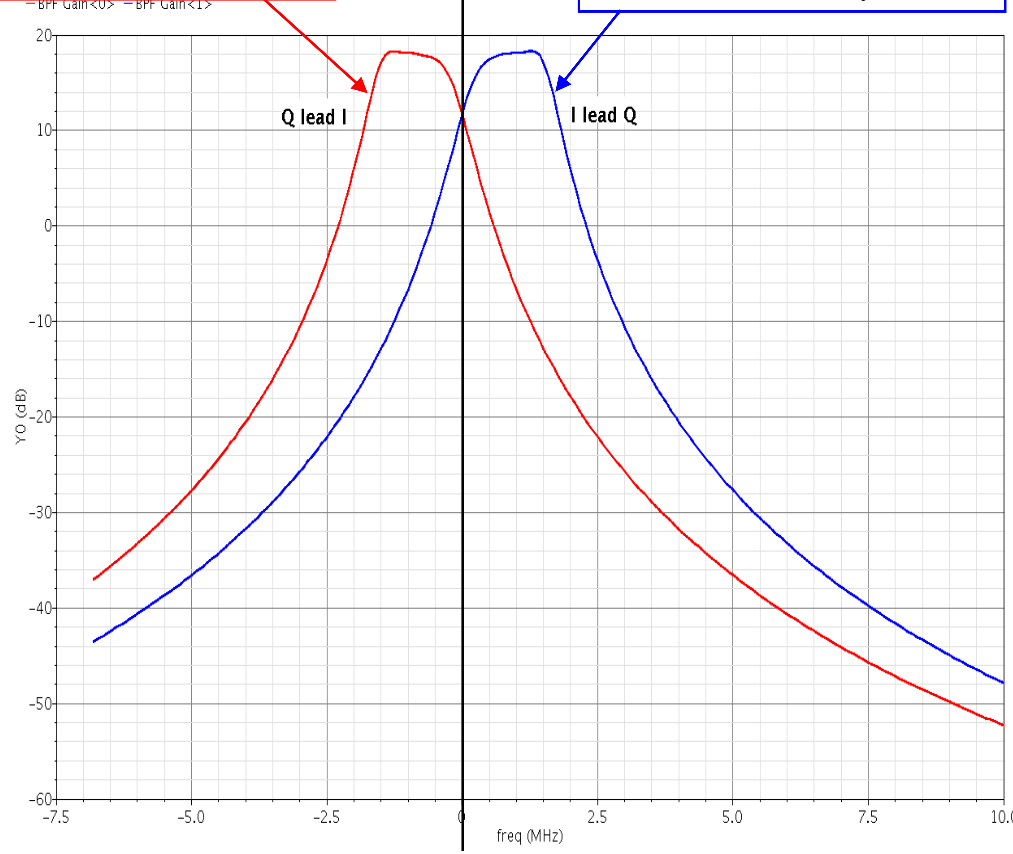


$$\omega_C = \frac{1}{R_C C}$$

$$\omega_0 = \frac{1}{R_1 C}$$

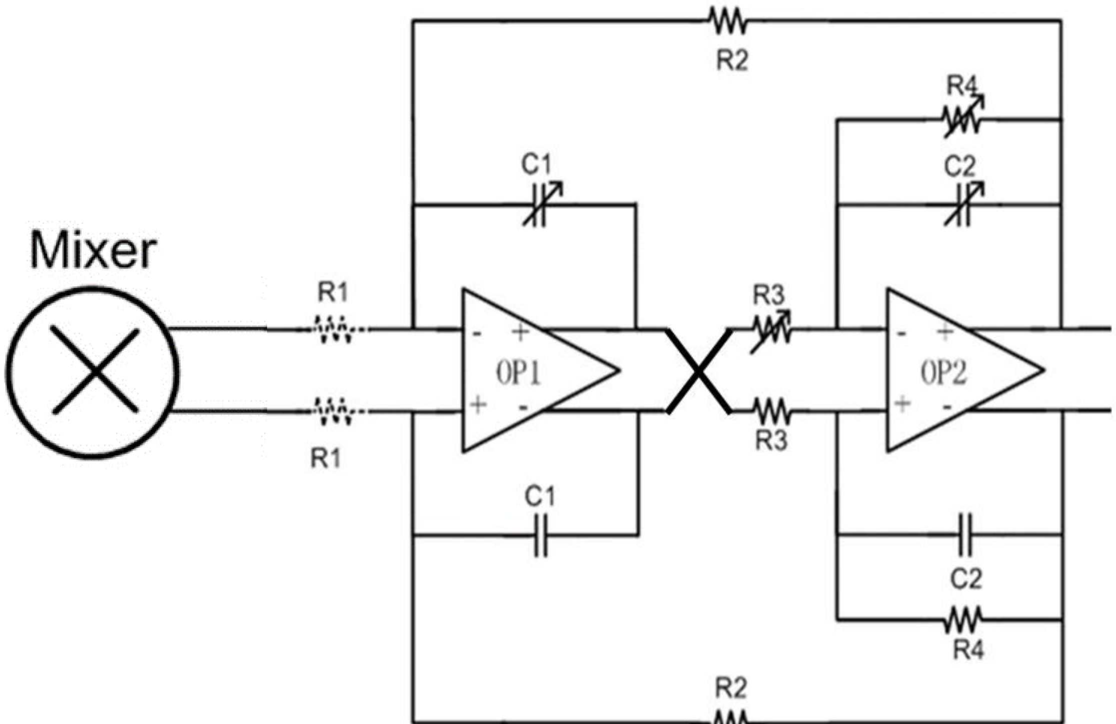
$$V_{out,I} = \frac{1}{1 + j\dfrac{\omega + \omega_c}{\omega_0}} \cdot V_{in,I}$$

$$V_{out,I} = \frac{1}{1 + j\dfrac{\omega - \omega_c}{\omega_0}} \cdot V_{in,I}$$



Note that you can decide to favor upper mix or lower mix in this low-IF receiver (meaning positive vs negative frequencies) by changing the sign of the cross-coupled resistor voltages (I leads Q or Q leads I). The not flat passband is due to finite opamp GBW

Complex cascaded filter realization:



$$A_v = \frac{R_2}{R_1}$$

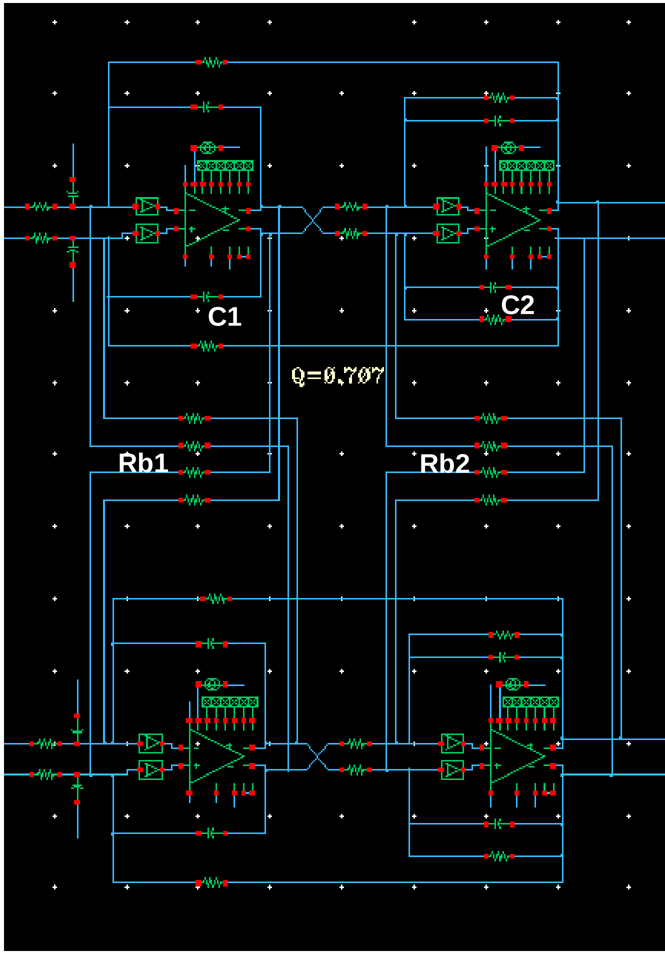
$$\omega_0 = \sqrt{\frac{1}{R_2 R_3 C_1 C_2}}$$

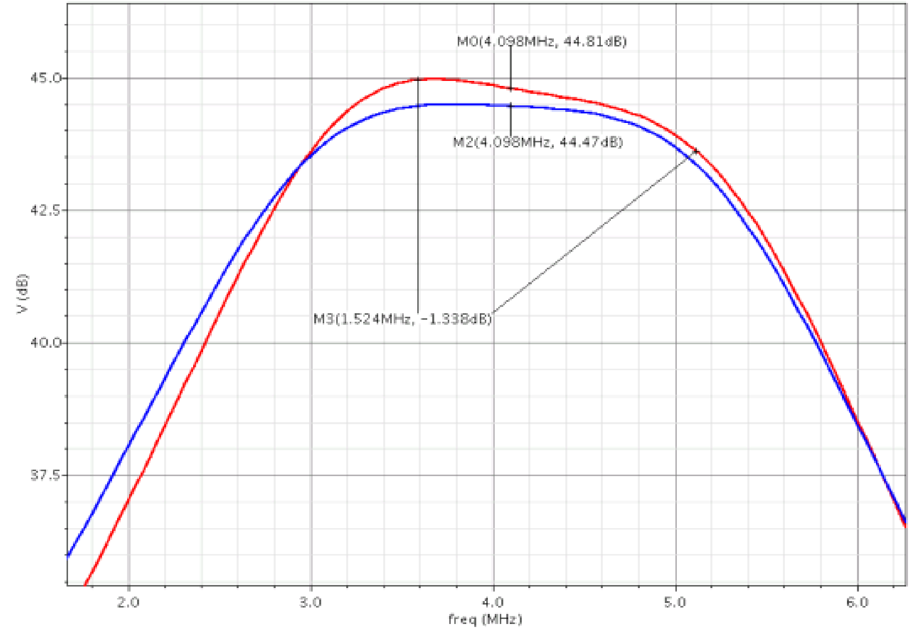
$$Q = R_4 \sqrt{\frac{C_2}{R_2 R_3 C_1}}$$

$$\omega_c = \frac{1}{R_{b1} C_1} = \frac{1}{R_{b2} C_2}$$

This is a biquad (real low pass filter). First you chose circuit parameters based on noise, linearity and signal BW. Next step is to design the cross coupled resistors between I and Q filters to create a complex filter as shown next

complex cascade filter:

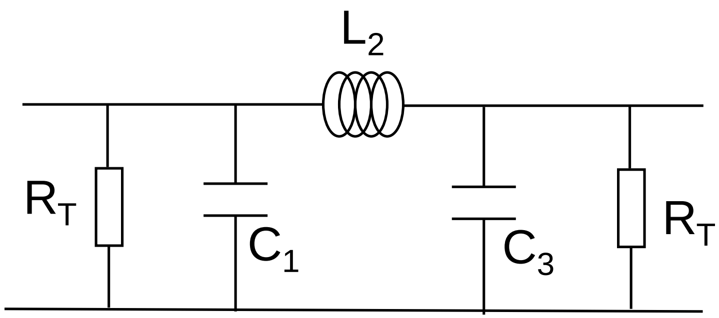




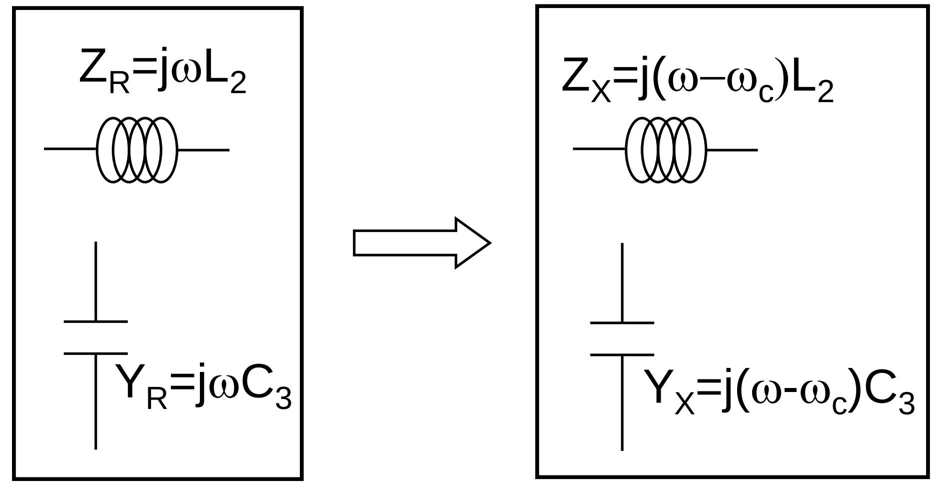
$$\omega_c = \frac{1}{R_{b1}C_1} = \frac{1}{R_{b2}C_2}$$

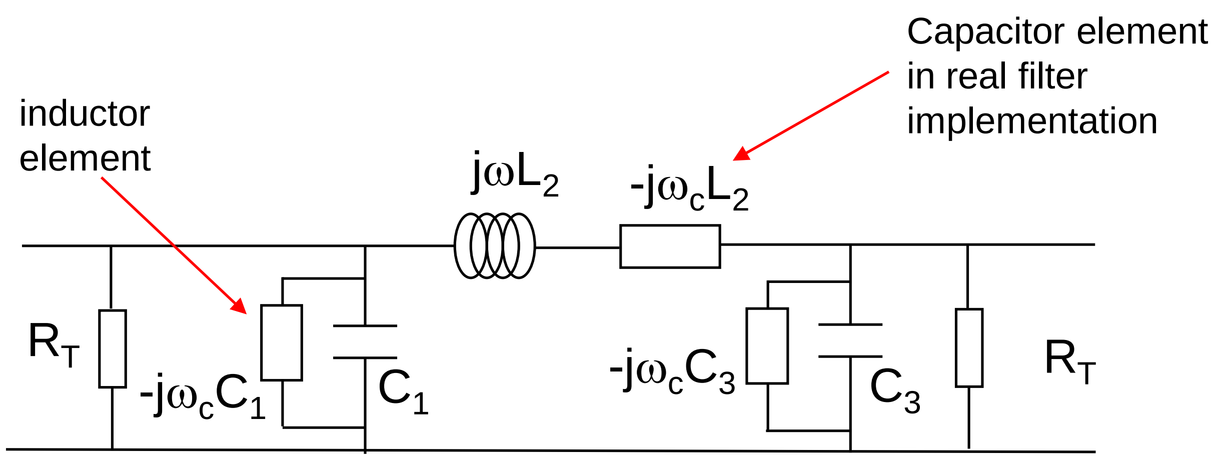
- Rb1 and Rb2 are calculated based on C1, C2 and ωc values. In this design, ωc is 4MHz
- Real opamp finite GBW causes some none flat passband response (red) vs ideal opamp (blue). Opamp compensation pole is not shifted!

Real to complex ladder filter realization:

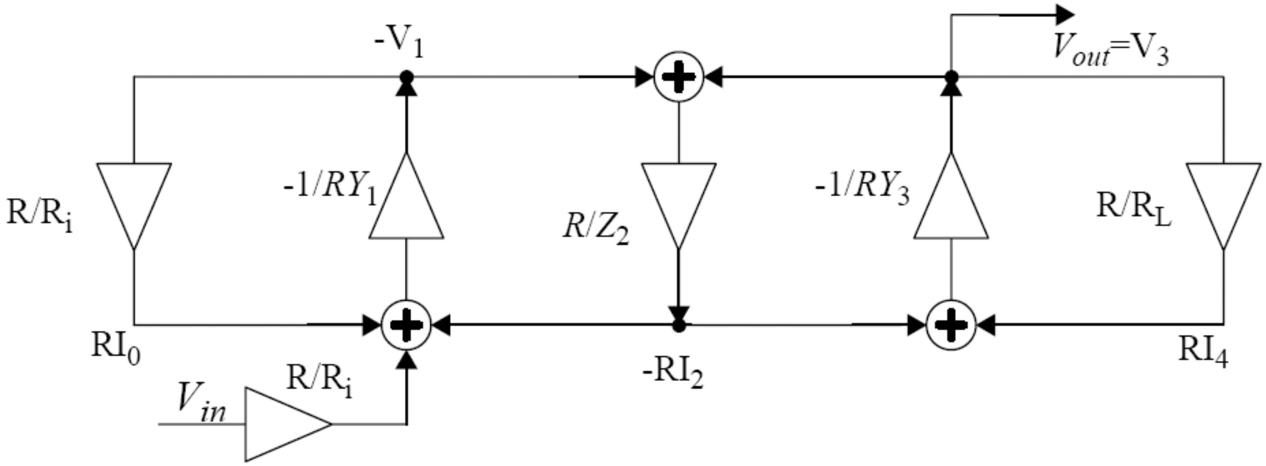


Real filter





complex filter



Let us take the example of calculating the complex SFG for L_2:

$$(V_3 - V_1)\frac{R}{Z_2} = -RI_2$$

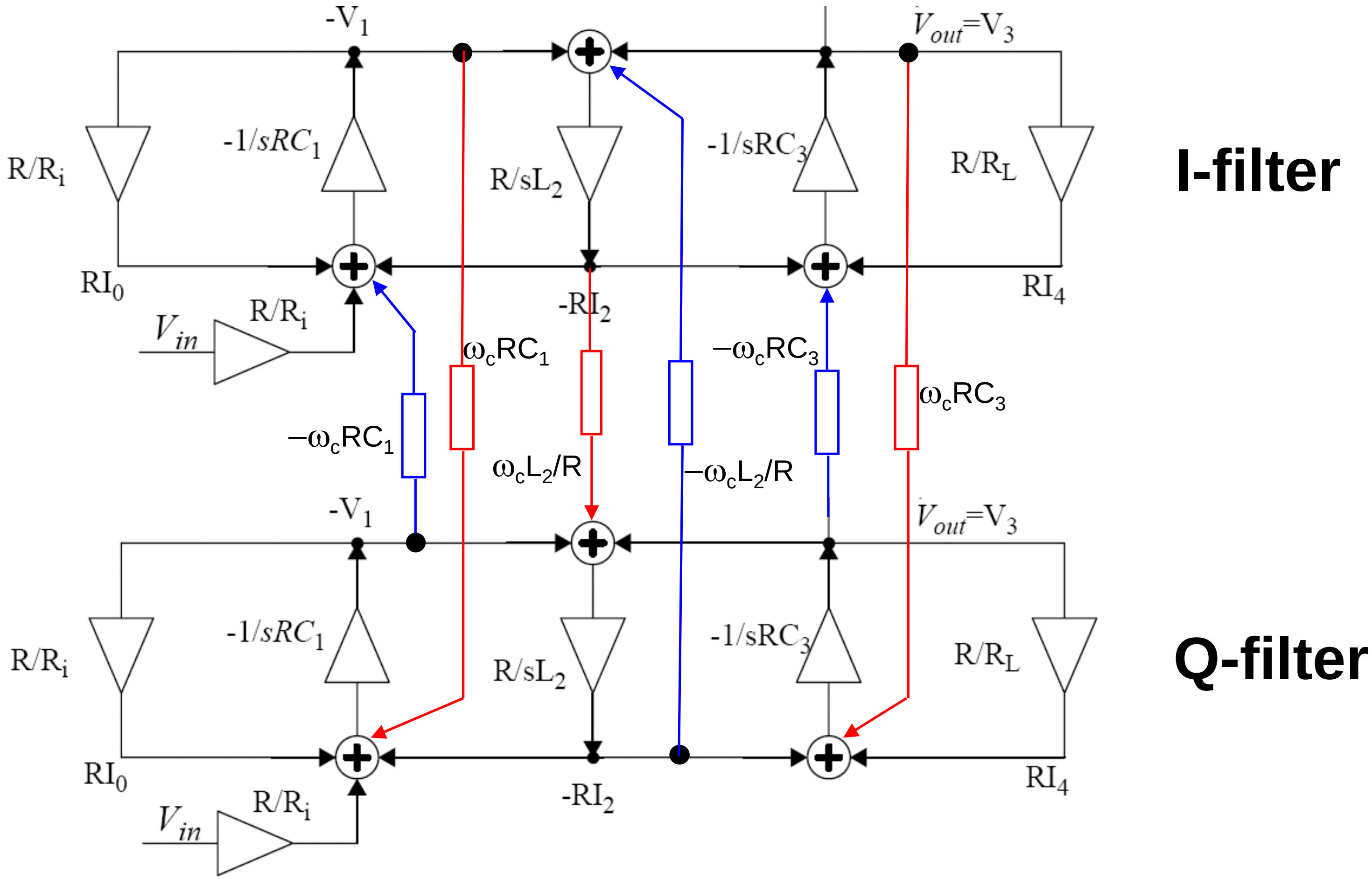
$$(V_3 - V_1)\frac{R}{j(\omega - \omega_c)L_2} = -RI_2$$

$$R(V_3 - V_1) = -j\omega L_2 RI_2 + j\omega_c L_2 RI_2$$

$$-RI_2 = \frac{R}{j\omega L_2}\left[(V_3 - V_1) - \frac{\omega_c L_2}{R}(jRI_2)\right]$$

The example analysis shows that the complex signal flow-graph is constructed by starting first with the real lowpass filter signal flow graph, then adding a coupling resistor from the output of each opamp of I channel to the input of the corresponding opamp in the Q filter and visa versa (with careful attention to the sign).

full complex ladder filter SFG:

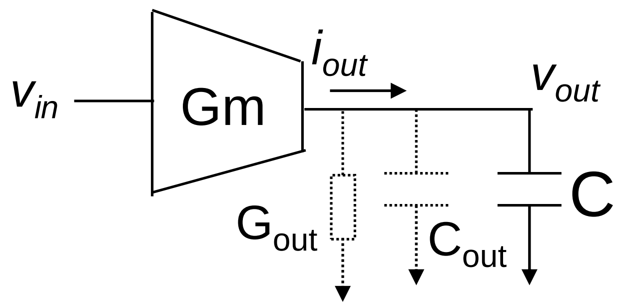


Only 3 integrators per filter are needed to implement this complex bandpass filter as compared to 6 integrators per filter for real bandpass filter implementation!

Appendix

Impact of nonideality of Gm cell on the filter frequency response:

1. Finite Gm output impedance:



The finite output conductance of the Gm cell can be modeled as a finite G_{out} in parallel with the integrating cap. Similarly, the parasitic output capacitance of the Gm cell appears is parallel with the integration capacitor. This means the ideally lossless integrator becomes a bit lossy with the following transfer function:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{g_m}{sC_T + G_{out}} \quad ; \; C_T = C + C_{out}$$

as seen, G_{out} appears as "loss" in the integrator transfer function. The value of C can be adjusted to "absorb" the parasitic capacitance C_{out}. This technique is called *predistortion*.

The finite G_{out} has an impact on the realizable biquad "Q" as follows. The biquad transfer function can be expressed in terms of natural frequency ω_n and quality factor Q (based on control theory) as:

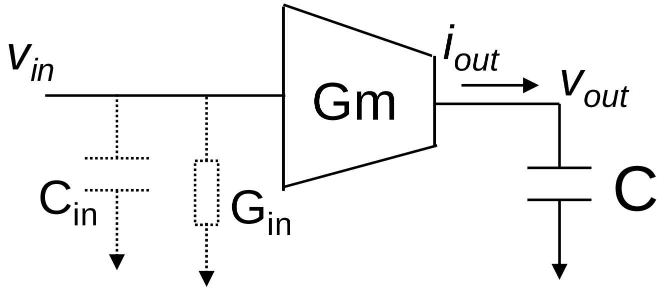
$$\frac{\omega_n^2}{s^2 + \frac{\omega_n}{2Q}s + \omega_n^2}$$

Filters with very sharp stopband roll off (close to unity stopband to passband ratio) have high Q biquads in their realization. Therefore, with finite G_{out} the lossless-lossy integrator biquad function can be rewritten as:

$$\frac{V_{out}}{V_{in}} = H(s) = \frac{T}{1+T} = \frac{\frac{A}{s+kG_{out}}\frac{B}{s+b}}{1+\frac{A}{s+kG_{out}}\frac{B}{s+b}} = \frac{AB}{s^2 + (b+kG_{out})s + (AB + bkG_{out})}$$

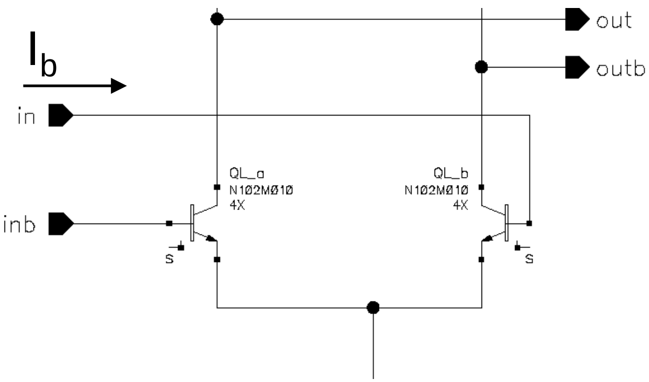
It can be seen that G_{out} increases the coefficient of the "s" term, indicating a lower Q. This means the maximum filter Q that can be realized by the design will be limited.

2. Finite Gm input impedance:

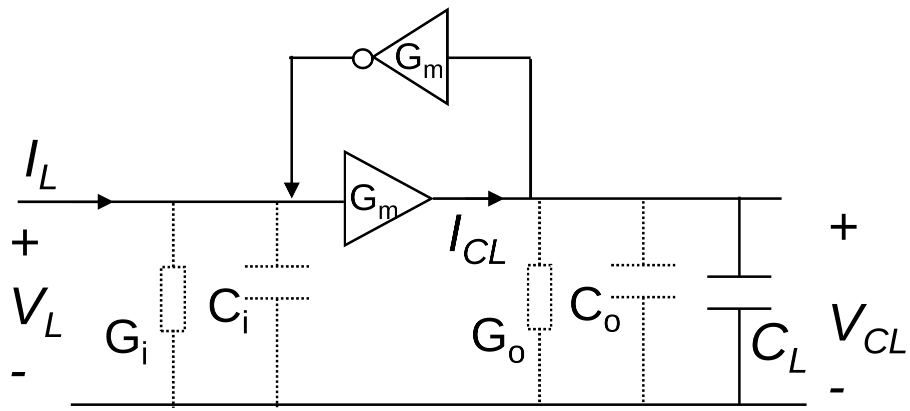


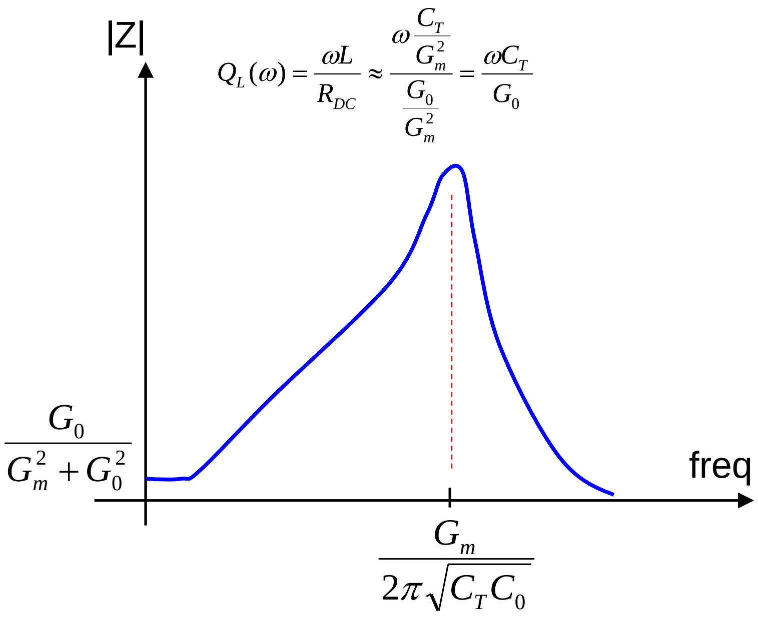
The finite input conductance, as well as parasitic input capacitance, of the Gm cell can be modeled as a finite G_{in} and C_{in} in parallel with the input. Since the filter realization is a cascade of biquads and integrators, the input of a Gm cell loads the output of the other resulting in the same impact on the transfer function as G_{out}, and C_{out}.

Bipolar-based Gm cell without degeneration suffers high input conductance. Although speed is high, such Gm cell cannot realize high Q filters (Q>5 or so)



Impact of Gm cell input/output parasitics on the Gyrator performance:





$$G_i = G_{in} + G_{out} = G_0 \quad ; \quad C_i = C_{in} + C_{out} = C_0$$

$$I_{C_L} = (G_0 + sC_T)V_{C_L} \quad ; \quad C_T = C_L + C_0$$

$$I_L - V_L(G_0 + sC_0) = G_m V_{C_L}$$

$$I_{C_L} = G_m V_L = (G_0 + sC_T)V_{C_L}$$

$$\Rightarrow V_L = \frac{G_0 + C_T s}{(G_m^2 + G_0^2) + G_0 s(C_T + C_0) + C_T C_0 s^2} I_L$$

$$\text{and} \quad Z = \frac{G_0 + C_T s}{(G_m^2 + G_0^2) + G_0 s(C_T + C_0) + C_T C_0 s^2}$$

3. Finite Gm cell bandwidth:

The finite Gm cell bandwidth can be modeled as a finite single-pole system as:

$$G_m(s) = \frac{G_{m0}}{\frac{s}{P_0}+1}$$

Where G_{m0} is the DC value and P_0 is the finite pole representing the finite bandwidth. When this Gm cell is used to build a biquad, the following transfer function results

$$H_{BQ}(s) = \frac{G_m(s)^2}{C_1C_2s^2 + G_m(s)C_1s + G_m(s)^2} = \frac{{G_{m0}}^2}{C_1C_2s^2\left(\frac{s}{P_0}+1\right)^2 + G_{m0}C_1s\left(\frac{s}{P_0}+1\right) + {G_{m0}}^2}$$

$$= \frac{1}{\frac{C_1C_2}{{G_{m0}}^2P_0^2}s^4 + \frac{2C_1C_2}{{G_{m0}}^2P_0}s^3 + \left(\frac{C_1C_2}{{G_{m0}}^2} + \frac{C_1}{G_{m0}P_0}\right)s^2 + \frac{C_1}{G_{m0}}s + 1}$$

As seen, the finite Gm cell bandwidth results in increasing the order of the biquad section. This impacts the overall filter frequency response if P_0 is too close to the filter bandwidth. A general practice is to chose the Gm cell bandwidth to be 10 times the maximum frequency of interest in the filter response for low to moderate Q filters.

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[1] R. Schaumann, K. Laker, M. Ghaussi, “Design of Analog Filters, Passive, Active RC, and Switched Capacitor,” Prentice Hall, 1990.

[2] J. Carlos, M. Steyaert, *CMOS Wireless Transceiver Design* , Kluwer Academic Publishers. 1997

[3] A. Sedra, W. Sneigrove, R. Allen, “Complex Analog Baseband Filters Designed by Linear Shifting Real Low-Pass Prototypes,” *in Proceedings of ISCAS 1985*, pp. 1223-1226.