1 Evaluation Criterion

The evaluation of the first homework was done as follows:

- 10 points were awarded for completion of the homework and the other 10 points were awarded for correctness of the arguments.
- In order to get full points for completion the student has to give a serious attempt of solution to each of the problems.
- Half of the problems were completely read by me (Ian) and I wrote very short comments when I found it necessary. Even if you got a perfect score I might have written something.
- The second half of the problems were superficially read until I was convinced that the student was making a serious attempt of solution. In some cases I found mistakes on serious attempts of proofs for those problems. In those cases I also wrote comments, but no points for correctness were deducted. For non-serious attempts of solution points for completeness were deducted.

2 List of common mistakes and some comments

- If G is a group and $H \leq G$ is a subgroup, then it is NOT always true that: $g_1 H g_2 H = g_1 g_2 H$. For this identity to hold we usually require H to be a normal subgroup of G.
- For problem 3, there was some confusion with the left action of G on left cosets. The following function $f : G \times G/H \to G/H$ is not well defined: $f(g, aH) = agH$. Suppose we have two representatives $aH = ahH$ for $f(g, aH)$ to be well defined for each g we have to verify that $ahgH = agH$. That is: $g^{-1}a^{-1}ahg \in H$ or equivalently $h \in gHg^{-1}$ has to hold for each $g \in G$. This would imply H is a normal subgroup of G, which was not in the hypothesis of this problem.
- For problem 2, one can not use the classification of finite abelian groups unless you prove beforehand that every group of order 4 is abelian.
- For problem 3 the common solution would construct a permutation action of G on the set G/H of left cosets. This would give a group homomorphism to S_n where $n = [G : H]$. The mistake was to assume that the map $f : G/Ker(f) \to S_n$ is an isomorphism. In the general case $G/Ker(f)$ is only a subgroup of S_n.
- For problem 4, there were attempts of solution that would map the left coset xH to the right coset Hx. This assignment will be well defined but it is not necessarily a bijection.
- The most common mistake happened for problem 3. One had to prove that if H is a subgroup of G of finite index, then there is a normal subgroup contained in H that also has finite index. Many people forgot to prove the finite index part.