Math 250a hw8

14.

- For (a) and (b), using the snake lemma will also make our lives easier. For example, (a) states that if $0 \to \ker g \to 0$ is exact, then $\ker g = 0$, which must be true.
- For (c), we may extend snake lemma to include $\operatorname{coker} g \to \operatorname{coker} h \to 0$ because $N \to N'' \to 0$ is assumed to be exact.

17.

- The p-adics are more than integers written in base p. There can be infinite strings.
- p is not a formal variable as x in $\mathbb{Z}/p\mathbb{Z}[\![x]\!]$, because there is a p-carry.
- To show surjectivity, we may use the surjections from $\mathbb{Z}$ to $\mathbb{Z}/p^i\mathbb{Z}$ and the universal property of $\mathbb{Z}_p$. Thus, this projection is in fact the composition $\mathbb{Z} \to \mathbb{Z}_p \to \mathbb{Z}/p^i\mathbb{Z}$, whence $\mathbb{Z}_p \to \mathbb{Z}/p^i\mathbb{Z}$ must be surjective.
- Inverse limit $\varprojlim$ can be typeset by `\varprojlim`. Directed limit $\varinjlim$ can be typeset by `\varinjlim`.
- Some asks about what to do after constructing a unique map from LHS to RHS and vice versa. I guess you can show they are mutually inverse. It manifests the universal property of $\varprojlim$ – "unique up to unique isomorphism".
- By the universal property of limit, limit commutes with limit. Product is a special kind of limit. It is a limit over a discrete diagram.

21.

- It would be nice if you may introduce your notation (denoting an element in the direct limit) at the beginning.
- Although it looks like you have to show three things, they are really the same. You can show the middle exactness and then let the left or right be 0 to obtain injectivity/surjectivity respectively.