1 List of common mistakes and some comments

- When on wants to say that two elements of a field $x, y \in K$ are algebraically independent they have to specify over which subfield of K they are algebraically independent.

- A lot of students did a lot of juggling in proving that in a characteristic $p > 0$ field k, a polynomial of the form $f(x) = x^p - a$ with $a \in k$ is either irreducible or has a root in k. The standard argument is the following: $\alpha \in \overline{k}$ satisfy $f(\alpha) = 0$ in some algebraic closure $\overline{k}/k$. We have that in $\overline{k}$, $f(x)$ factors as $(x-\alpha)^p$. Let $g(x)$ be the minimal polynomial of α. Then $g(x)$ divides $f(x)$ and is of the form $(x-\alpha)^m$ with $m \leq p$. If $1 < m < p$ then $g'(x) \neq 0$ and $g'(\alpha) = 0$ which contradicts the fact that $g(x)$ is the minimal polynomial of α. This proves $m = 1$ or $m = p$ which correspond precisely to the case in which $f(x)$ has a root or $f(x)$ is irreducible respectively.

- It is not true that finite fields have a finite number of irreducible polynomials. Indeed, any element $\alpha \in \overline{\mathbb{F}_q}$ defines a unique irreducible monic polynomial $min_\alpha(x) \in \mathbb{F}_q[x]$. Since each irreducible polynomial has a finite number of roots in an algebraic closure it is enough to prove that any algebraic closure of a finite field is infinite. We prove that finite fields are not algebraically closed. The units of a finite field of order q form a cyclic group of order $q-1$. For any d dividing $q-1$ the function $x^d : \mathbb{F}_q^\times \to \mathbb{F}_q^\times$ is not injective and consequently not surjective. In particular the equation $x^d - a$ does not have a root for some $a \in \mathbb{F}_q$, which proves this field is not algebraically closed.

- When proving $k(t^p, u^p) \subseteq k(t, u)$ has an infinite number of intermediate field extensions many students produced an infinite family of elements $\alpha \in k(t, u)$ for which the extensions $k(t^p, u^p, \alpha)$ was always the same extensions. One has to remember then that many different elements can define the same field extension. For example $\mathbb{Q}(\sqrt{2}) = \mathbb{Q}(\sqrt{2^{2n+1}})$